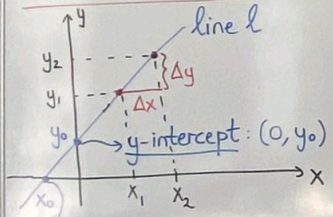
Math 113 - Exam dates:

Midterm 1: Nov. 17, 2022 – 17:30 (Thursday)

Midterm 2: Dec. 22, 2022 – 17:30 (Thursday)

Lines on the Plane:



x-intercept: $(x_0, 0)$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \rightarrow \text{slope of line } \ell$$

eqn(s) of line ℓ:

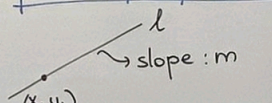
* slope – intercept eqn. of ℓ:

$$y = mx + n$$

n: $(0, n) \rightarrow$ y-intercept of ℓ

m: slope of ℓ

* point-slope eqn. of ℓ:

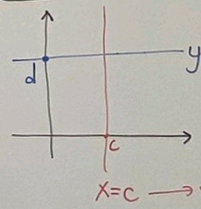


$$y - y_1 = m(x - x_1)$$

* $\ell_1 \parallel \ell_2$: lines ℓ_1 & ℓ_2 are parallel $\Leftrightarrow$ $m_1 = m_2$ $\rightarrow$ slope of ℓ_2 = slope of ℓ_1

* $\ell_1 \perp \ell_2$: lines ℓ_1 & ℓ_2 intersect at 90° (i.e. ℓ_1 & ℓ_2 are orthogonal (or perpendicular))

$\Leftrightarrow$ $m_1 \cdot m_2 = -1$, (i.e. $m_1 = \frac{-1}{m_2}$)



$y = d \rightarrow$ horizontal line

$x = c \rightarrow$ vertical line

Standard line eqn:

$$ax + by = c$$

HW: Redo examples 1.6, 1.7, 1.8, 1.9 on page 7

p.10/1.31 Find the eqns. of the following lines:

Passes through the origin $((0,0))$ and has slope $m = \frac{1}{5}$.

$$y - 0 = \frac{1}{5}(x - 0) \Rightarrow y = \frac{1}{5}x \Rightarrow 5y = x \Rightarrow \boxed{x - 5y = 0}$$

1-34) line l_1 passes through $(0,-3)$ and parallel to the line $\underbrace{10y-5x=99}_{l_2}$

$l_1 \parallel l_2 \Rightarrow m_1 = m_2$

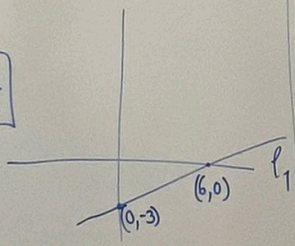
$l_2: 10y = 5x + 99$

$y = \frac{5}{10}x + \frac{99}{10} = \left(\frac{1}{2}\right)x + \frac{99}{10}$

m_2: slope of $l_2 = \boxed{\frac{1}{2}}$

$\Rightarrow l_1 \parallel l_2 \Rightarrow m_1 = m_2 = \boxed{\frac{1}{2}}$

$(0,-3)$ on $l_1 \Rightarrow$



$y-(-3) = \left(\frac{1}{2}\right)(x-0)$

$\boxed{y = \frac{1}{2}x - 3}$ ($(0,-3) \rightarrow$ y-intercept of l_1)

1-35) l_1 passes through $(9,12)$ and perpendicular to the line l_2: $2x+5y=60$

$l_1 \perp l_2 \Rightarrow \boxed{m_1 = \frac{-1}{m_2}}$

$l_2: 2x+5y=60$

$5y = -2x + 60$

$\boxed{y = \left(\frac{-2}{5}\right)x + 12}$

$m_2 = -\frac{2}{5} \Rightarrow m_1 = \frac{-1}{m_2} = \boxed{\frac{5}{2}}$

l_1: $(9,12) \in l_1$

$m_1 = \frac{5}{2}$

$y-12 = \left(\frac{5}{2}\right)(x-9)$

$y = \frac{5}{2}x - \frac{45}{2} + 12 = \frac{5}{2}x - \frac{23}{2}$

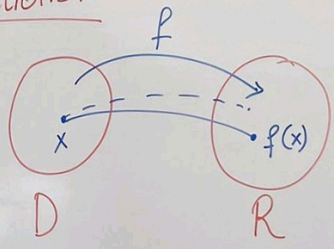
l_1:

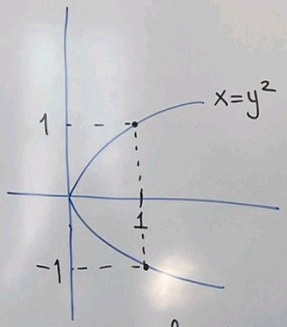
$2y = 5x - 23$

$\boxed{5x-2y=23}$ $(ax+by=c)$

$\underbrace{\qquad}_{l_1}$

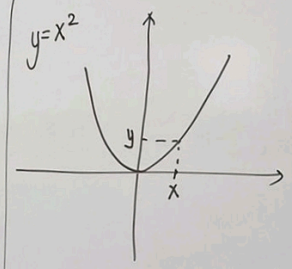
Functions:





not a function

$(-1)^2 = 1$, $(+1)^2 = 1$



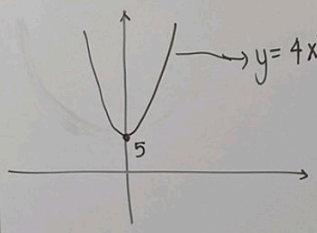
D: the set of all reals for which $f(x)$ is defined is called the domain of $f(x)$.

Range: The set of all values $f(x)$ is called the range of $f(x)$.

eg.: $x^2 \geq 0 \Rightarrow 4x^2 \geq 0 \Rightarrow 4x^2 + 5 \geq 5$

$y = 4x^2 + 5 \geq 5 \Rightarrow$ Range: $[5, \infty)$

Domain: $(-\infty, \infty)(\mathbb{R})$



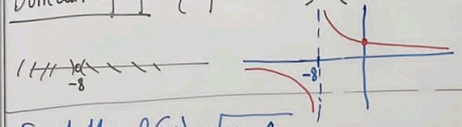
$f(x) = ax + b \rightarrow$ linear eqn.

$D = (-\infty, \infty)$

$R = (-\infty, \infty)$

Ex. 1-10: Find the domain of $f(x)=\frac{1}{x+8}$ hyperbola

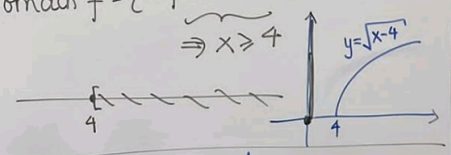
Domain of f: $\{x \mid x \neq -8\} = (-\infty,-8)\cup(-8,\infty)$



Ex. 1-11: $f(x)=\sqrt{x-4}$

Domain $f = \{x \mid x-4 \geq 0\} = [4,\infty)$

$\Rightarrow x \geq 4$



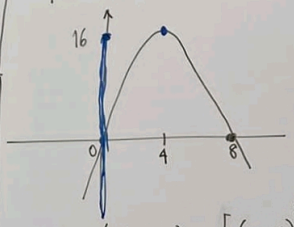
Ex. 1-12: $g(x)=\frac{1}{\sqrt{x-4}}$

Domain $g(x)=\{x \mid x-4>0\} = (4,\infty)$

p. 10/1-38 Find the domain and range of the following:

$f(x)=8x-x^2=x(8-x)$

$f(x)=0 \Rightarrow x=0$ or $x=8$



$f(x)=-(x^2-8x)=-[(x-4)^2-16]$

$f(x)=-(x-4)^2+16$

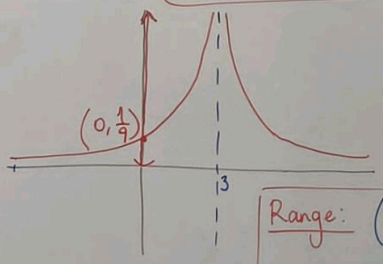
Domain $f(x)$: $(-\infty,\infty)$

Range $f(x)$: $(-\infty,16]$

1-39) $f(x)=\frac{1}{x^2-6x+9}=\frac{1}{(x-3)^2}$

f is not defined at $x=3 \Rightarrow$

Domain f: $(-\infty,3)\cup(3,\infty)$



Range: $(0,\infty)$ ↑ y-values

y-intercept: $(0,\frac{1}{9})$

x-intercept: no x-intercept(s)