Ch1. Functions:

Equations:

e.g. $8x-2=5x+7$

$8x-5x=7+2$

$\frac{3x}{3}=\frac{9}{3}$

$\boxed{ax+b=c}$ linear function

$\boxed{x=3}$

Ex. 1.1: (rational function)

$\frac{2x}{2x+5}=\frac{3}{4}$ $\left[x \neq -\frac{5}{2}\right]$

$4(2x)=3(2x+5)$

$8x=6x+15$

$2x=15 \Rightarrow \boxed{x=\frac{15}{2}}$

Soln: $\left\{\frac{15}{2}\right\}$

Absolute value:

$|-3|=3$, $|3|=3$

$$\boxed{|x|=\begin{cases} x, & x \geq 0 \\ -x, & x<0 \end{cases}}$$

Ex. 1-2: $|3x-12|=27$

$\Rightarrow (3x-12)=27$ or $-(3x-12)=27$

($3x>12 \Rightarrow x>4$) ✓

$3x=27+12=39$

$\boxed{x=13}$

($3x<12 \Rightarrow x<4$) ✓

$-3x+12=27$

$-3x=27-12=15$

$\boxed{x=-5}$

$\boxed{\text{Soln: } \{-5, 13\}}$

p10/1-23) $\left(\sqrt{x^2+16}\right)^2=(5)^2$ (Solve the equation $\Rightarrow$ i.e. find the value of the unknown x)

$x^2+16=25$

$x^2=25-16=9$

$\Rightarrow \boxed{x=\mp 3}$

Alternatively:

$\sqrt{x^2+16}=\sqrt{25}=\mp 5$

$$\sqrt{x^2}=\begin{cases} x \\ -x \end{cases}=|x|$$

$$\sqrt{x^2}=|x|$$

1-24) $|x-2|=12$

$x>2 \Rightarrow |x-2|=x-2$

$x<2 \Rightarrow |x-2|=-(x-2)$

$x-2=12$, $\boxed{x=14}$

$-(x-2)=12$

$x-2=-12$

$x=-12+2=-10$

$\boxed{x=-10}$

Soln.: $\{-10, 14\}$

Review of Algebra: (p. 9)

1-2) $\left(\frac{1}{16}\right)^{3/4}=\frac{(1)^{3/4}}{(2^4)^{3/4}}=\frac{1}{2^{\cancel{4}\cdot\frac{3}{\cancel{4}}}}=\frac{1}{2^3}=\boxed{\frac{1}{8}}$

$(a)^{\frac{m}{n}}=\sqrt[n]{a^m}=\left(\sqrt[n]{a}\right)^m$

1-5) $\sqrt[3]{\frac{8}{1000}}=\frac{(2^3)^{1/3}}{[(10)^3]^{1/3}}=\frac{2}{10}=\boxed{\frac{1}{5}}$

1-7) $(a+b)^2=a^2+2ab+b^2$

$(a-b)^2=a^2-2ab+b^2$

1

1 2 1

1 3 3 1

1 4 6 4 1

$(a+b)^3=a^3+3a^2b+3ab^2+b^3$

$(a-b)^3=[a+(-b)]^3$

$=a^3+3a^2(-b)+3a(-b)^2+(-b)^3$

$=\boxed{a^3-3a^2b+3ab^2-b^3}$

$a^2-b^2=(a-b)(a+b)$

$a^3+b^3=?\quad a^3-b^3=?$

$$\begin{array}{l|l} a^3+b^3 & a+b \\ \underline{-a^3+a^2b} & a^2-ab+b^2 \\ -a^2b+b^3 & \\ \underline{+a^2b+ab^2} & \\ ab^2+b^3 & \\ \underline{-ab^2+b^3} & \\ 0 & \end{array} \Rightarrow \boxed{a^3+b^3=(a+b)(a^2-ab+b^2)}$$

$$\begin{array}{l|l} a^3-b^3 & a-b \\ \underline{-a^3+a^2b} & a^2+ab+b^2 \\ a^2b-b^3 & \\ \underline{-a^2b+ab^2} & \\ ab^2-b^3 & \\ \underline{-ab^2+b^3} & \\ 0 & \end{array} \Rightarrow \boxed{a^3-b^3=(a-b)(a^2+ab+b^2)}$$

1-13) $x^3-1=(x-1)(x^2+x+1)$

$\lim_{x\to 1}\left(\frac{x^3-1}{x-1}\right)=\frac{0}{0}$!! ?

$=\lim_{\substack{x\to 1\\(x\neq 1)}}\left[\frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{(x-1)}}\right]=1^2+1+1=\boxed{3}$

$$a^4-b^4=(a^2)^2-(b^2)^2=(a^2-b^2)(a^2+b^2)$$
$$=(a-b)(a+b)(a^2+b^2)$$

$$a-b=(\sqrt{a})^2-(\sqrt{b})^2=(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})$$

1-9) $$\frac{1}{(\sqrt{5}-\sqrt{3})}\cdot\frac{(\sqrt{5}+\sqrt{3})}{(\sqrt{5}+\sqrt{3})}=\frac{\sqrt{5}+\sqrt{3}}{5-3}=\frac{\sqrt{5}+\sqrt{3}}{2}$$
$$=\boxed{\frac{\sqrt{5}}{2}+\frac{\sqrt{3}}{2}}$$

1-10) $$\boxed{\frac{12}{\sqrt{7}-1}-\frac{12}{\sqrt{7}+1}=4}$$
$(\sqrt{7}+1)$ $(\sqrt{7}-1)$
$$=\frac{12(\sqrt{7}+1)-12(\sqrt{7}-1)}{(\sqrt{7}+1)(\sqrt{7}-1)}=\frac{\cancel{12\sqrt{7}}+12-\cancel{12\sqrt{7}}+12}{6}$$
$(7-1)=6$
$$=\frac{24}{6}=\boxed{4}$$

1-12) $$\sqrt{x^3y}\,\sqrt{64xy^9}$$
$$\left[\sqrt[n]{a}\,\sqrt[n]{b}=\sqrt[n]{ab} \qquad (a)^{\frac{r}{s}}(a)^{\frac{p}{q}}=a^{\frac{r}{s}+\frac{p}{q}}\right]$$
$$=\sqrt{x^3y\cdot 64xy^9}$$
$$=\sqrt{64x^4y^{10}}=(8^2)^{1/2}\cdot(x^4)^{1/2}\cdot(y^{10})^{1/2}=(8)(x^2)(y^5)$$
$$=\boxed{8x^2y^5}$$

1-14) $$\underbrace{(\sqrt{x^2+4}+3)}_{(a+b)}\underbrace{(\sqrt{x^2+4}-3)}_{(a-b)}=(a^2-b^2)$$
$$=(\sqrt{x^2+4})^2-(3)^2$$
$$=(x^2+4)-9=\boxed{x^2-5}$$

1-15) $$x^4-100y^4=(x^2)^2-(10y^2)^2$$
$$=\underbrace{(x^2-10y^2)}_{[(x)^2-(\sqrt{10}y)^2]}(x^2+10y^2)=\boxed{(x-\sqrt{10}y)(x+\sqrt{10}y)(x^2+10y^2)}$$

1-17) $$(3a-2b)^2=(3a)^2-2(3a)(2b)+(2b)^2$$
$$=\boxed{9a^2-12ab+4b^2}$$

1-19) $$\frac{2x}{x^2-4}+\frac{5}{x+2}=\frac{2x+5(x-2)}{x^2-4}=\frac{2x+5x-10}{x^2-4}$$
$(x-2)(x+2)$ $(x-2)$
$$=\boxed{\frac{7x-10}{x^2-4}}$$

1-20) $$1-\frac{1}{1+\frac{1}{x}}=1-\frac{1}{\frac{x+1}{x}}=1-\frac{x}{x+1}=\frac{\cancel{x}+1-\cancel{x}}{x+1}$$
$$\left(\frac{a}{\frac{b}{c}}=a\cdot\frac{c}{b}\right)$$
$$=\boxed{\frac{1}{x+1}}$$

Intervals:

* Closed interval: $[a,b]=\{x \mid a\leq x\leq b\}$
* Open interval: $(a,b)=\{x \mid a<x<b\}$
* Half-open interval: $(a,b]=\{x \mid a<x\leq b\}$

  $[a,b)=\{x \mid a\leq x<b\}$
* Unbounded intervals:

  $(-\infty,a)=\{x \mid x<a\}$

  $(a,\infty)=\{x \mid x>a\}$

  $(-\infty,\infty)=\mathbb{R}$ (real line)

0

Ex 1-3: Solve the inequality

$$7x-5\leq 30$$

$\Rightarrow 7x\leq 30+5=35$

$\frac{7x}{7}\leq\frac{35}{7} \Rightarrow \boxed{x\leq 5}$

$\Rightarrow \boxed{(-\infty,5]}$

Ex. 1-4: $|a|<b \Rightarrow -b<a<b$

$-b \quad 0 \quad b$, a

Solve: $|x+10|<11$

$\Rightarrow -11<x+10<11$

$-11-10<x+10-10<11-10$

$\boxed{-21<x<1}$

$\Rightarrow$ Soln.: $(-21,1)$

Ex. 1-5: $|a|>b \Rightarrow a>b$ or $-a<-b$

$-b=-10 \quad 0 \quad b=10$

$|x+10|>11 \Rightarrow x+10>11$ or $\underbrace{-(x+10)>11}$

$\Downarrow$

$x>11-10=1$ $\quad$ $x+10<-11$

$\boxed{x>1}$ $\quad$ $x<-11-10$

$\boxed{x<-21}$

$-21 \quad 1$

Soln.: $(-\infty,-21)\cup(1,\infty)$ (union of disjoint intervals)

$\mathbb{R}\setminus[-21,1]$

1-28) $|12-7x|\geq 1 \Rightarrow \underbrace{12-7x\geq 1}$ or $-(12-7x)\geq 1$

$\underbrace{12-7x\leq -1}$

$\Downarrow$

$12-1\geq 7x$ $\quad$ $12+1\leq 7x$

$11\geq 7x$ $\quad$ $13\leq 7x \Rightarrow \frac{13}{7}\leq x$

$\boxed{x\leq\frac{11}{7}}$ $\quad$ $\Rightarrow \boxed{x\geq\frac{13}{7}}$

$\frac{11}{7} \quad \frac{13}{7}$

Soln.:

$\left(-\infty,\frac{11}{7}\right]\cup\left[\frac{13}{7},\infty\right)$