HW: problems

4-14)

Indeterminate forms:

$\frac{0}{0}, \frac{\infty}{\infty}, \infty - \infty, \frac{1}{\infty - \infty}$

$$\lim_{x \to \mp\infty} \left(\frac{c}{x^p}\right) = 0 \quad , p > 0, \ c \to \text{constant}$$

$$\lim_{x \to \mp\infty} \left(\frac{p(x)}{q(x)}\right) = \begin{cases} \mp\infty, & \text{if } \deg p(x) > \deg q(x) \\ * & \text{, if } \deg p(x) = \deg q(x) \\ 0, & \text{if } \deg p(x) < \deg q(x) \end{cases}$$

$*$ ratio of leading terms coeffs. of $p(x)$ & $q(x)$

$$\sqrt{x^2} = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

p 33/4.15

$$\lim_{x\to\infty}\left(\sqrt{x^2+4x}-\sqrt{x^2-10x+1}\right)=\infty-\infty$$

$$=\lim_{x\to\infty}\frac{\left(\sqrt{x^2+4x}-\sqrt{x^2-10x+1}\right)\left(\sqrt{x^2+4x}+\sqrt{x^2-10x+1}\right)}{\left(\sqrt{x^2+4x}+\sqrt{x^2-10x+1}\right)}$$

$$=\lim_{x\to\infty}\left[\frac{(\cancel{x^2}+4x)-(\cancel{x^2}-10x+1)}{\sqrt{x^2\left(1+\frac{4}{x}\right)}+\sqrt{x^2\left(1-\frac{10}{x}+\frac{1}{x^2}\right)}}\right]$$

$$\sqrt{x^2}=x$$

$$=\lim_{x\to\infty}\left[\frac{\cancel{x}\left(14-\underbrace{\frac{1}{x}}_{0}\right)}{\cancel{x}\left(\sqrt{1+\underbrace{\frac{4}{x}}_{0}}+\sqrt{1-\underbrace{\frac{10}{x}}_{0}+\underbrace{\frac{1}{x^2}}_{0}}\right)}\right]$$

$$=\frac{14-0}{\sqrt{1+0}+\sqrt{1-0+0}}=\frac{14}{2}=\boxed{7}$$

4.17)

$$\lim_{x\to\infty}\left(\sqrt{x^2-12x+24}-\sqrt{x^2+10x+5}\right)=\infty-\infty$$

$$=\lim_{x\to\infty}\frac{\left(\sqrt{x^2-12x+24}-\sqrt{x^2+10x+5}\right)\left(\sqrt{x^2-12x+24}+\sqrt{x^2-10x+5}\right)}{\left(\sqrt{x^2-12x+24}+\sqrt{x^2+10x+5}\right)}$$

$$=\lim_{x\to\infty}\frac{\underbrace{(\cancel{x^2}-12x+24)-(\cancel{x^2}+10x+5)}_{=-22x+19}}{x\left[\sqrt{1-\frac{12}{x}+\frac{24}{x^2}}+\sqrt{1+\frac{10}{x}+\frac{5}{x^2}}\right]}$$

$$=\lim_{x\to\infty}\frac{\cancel{x}\left[-22+\underbrace{\frac{19}{x}}_{0}\right]}{\cancel{x}\left(\sqrt{1-\underbrace{\frac{12}{x}}_{0}+\underbrace{\frac{24}{x^2}}_{0}}+\sqrt{1+\underbrace{\frac{10}{x}}_{0}+\underbrace{\frac{5}{x^2}}_{0}}\right)}$$

$$=\frac{-22}{\sqrt{1}+\sqrt{1}}=\frac{-22}{2}=-11$$

4.18.

$$\lim_{x\to\infty}\left(\frac{1}{2x-\sqrt{4x^2-5x+6}}\right)=\frac{1}{\infty}$$

$$=\lim_{x\to\infty}\frac{\left(2x+\overbrace{\sqrt{4x^2-5x+6}}^{x\sqrt{4-\frac{5}{x}+\frac{6}{x^2}}}\right)}{\left(2x-\sqrt{4x^2-5x+6}\right)\left(2x+\sqrt{4x^2-5x+6}\right)}$$

$$=\lim_{x\to\infty}\frac{x\left[2+\sqrt{4-\frac{5}{x}+\frac{6}{x^2}}\right]}{\underbrace{\cancel{4x^2}-(\cancel{4x^2}-5x+6)}_{(5x-6)=x\left(5-\frac{6}{x}\right)}}$$

$$=\lim_{x\to\infty}\frac{\cancel{x}\left[2+\sqrt{4-\underbrace{\frac{5}{x}}_{0}+\underbrace{\frac{6}{x^2}}_{0}}\right]}{\cancel{x}\left[5-\underbrace{\frac{6}{x}}_{0}\right]}$$

$$=\frac{2+\sqrt{4-0+0}}{5-0}=\frac{2+2}{5}=\boxed{\frac{4}{5}}$$

4.16) $\lim_{x\to\infty}\left[\frac{x^4-16}{\underbrace{(2x-1)(2x+1)}_{(4x^2-1)}(x^2+1)}\right] = \lim_{x\to\infty}\frac{(x^4-16)}{(4x^4+3x^2-1)}$

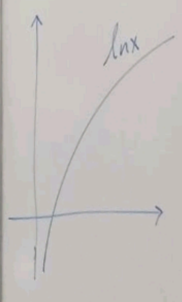
$(4x^2-1)(x^2+1)$

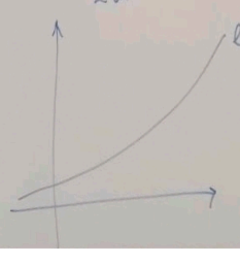
$4x^4+3x^2-1$

$$=\lim_{x\to\infty}\frac{\cancel{x^4}\left[1-\underbrace{\frac{16}{x^4}}_{0}\right]}{\cancel{x^4}\left[4+\underbrace{\frac{3}{x^2}}_{0}-\underbrace{\frac{1}{x^4}}_{0}\right]} = \boxed{\frac{1}{4}}$$

4.38) $\lim_{x\to\infty}\frac{1}{\ln(x^2)} = \frac{1}{\infty} = 0$

$$\lim_{x\to 0^+}\frac{1}{\underbrace{\ln(x^2)}_{2\ln x}} = \frac{1}{-\infty} = \boxed{0}$$





4-39) $\lim_{x\to\infty}\left(\frac{8e^x}{1+5e^x}\right) = \frac{\infty}{\infty}$

$$=\lim_{x\to\infty}\frac{\cancel{e^x}(8)}{\cancel{e^x}\left(\underbrace{\frac{1}{e^x}}_{\frac{1}{\infty}=0}+5\right)} = \frac{8}{0+5} = \boxed{\frac{8}{5}}$$

eg. $\lim_{x\to\infty}\left(\frac{+e^x}{1-3e^{2x}}\right) = \frac{\infty}{\infty}$

$$=\lim_{x\to\infty}\frac{\cancel{e^x}}{\cancel{e^x}\left(\underbrace{\frac{1}{e^x}}_{0}-3\underbrace{e^x}_{\infty}\right)}$$

$$=\frac{1}{0-\infty} = \frac{1}{-\infty} = \boxed{0}$$

HW: problems

4-14)

Indeterminate forms:

$\frac{0}{0}, \frac{\infty}{\infty}, \infty-\infty, \frac{1}{\infty-\infty}$

$\lim_{x\to\mp\infty}\left(\frac{c}{x^p}\right) = 0$, $p>0$, $c\to$ constant

$$\lim_{x\to\mp\infty}\left(\frac{p(x)}{q(x)}\right) = \begin{cases} \mp\infty, & \text{if } \deg p(x) > \deg q(x) \\ (*) & \text{if } \deg p(x) = \deg q(x) \\ 0, & \text{if } \deg p(x) < \deg q(x) \end{cases}$$

(*) ratio of leading terms coeffs. of $p(x)$ & $q(x)$

$$\sqrt{x^2} = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x<0 \end{cases}$$

One-sided Limits:

$\lim\limits_{x\to a^+} f(x)$: right-limit of $f(x)$ as x approaches "a" from values larger than "a"

$(x>a)$

a ←

$\lim\limits_{x\to a^-} f(x)$: left-limit of $f(x)$ as x approaches "a" from values less than "a"

$(x<a)$

→ a

Thm.: $\lim\limits_{x\to a} f(x)=L$ exists

$$\Longleftrightarrow \lim_{x\to a^-} f(x)=L=\lim_{x\to a^+} f(x)$$

p.38/ Ex.5.2 $\lim\limits_{x\to 3^+} f(x)=?$, $\lim\limits_{x\to 3^-} f(x)=?$

$$f(x)=\frac{4x-12}{|x-3|}$$

$$|x-3|=\begin{cases} x-3, & x>3 \\ -(x-3), & x<3 \end{cases}$$

$$\lim_{\substack{x\to 3^-\\(x<3)}} f(x)=\lim_{\substack{x\to 3^-\\x\neq 3\\(x<3)}}\left[\frac{4\cancel{(x-3)}}{-\cancel{(x-3)}}\right]=\frac{4}{-1}=\boxed{-4}$$

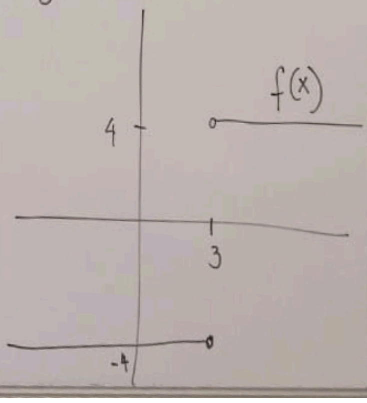
→3

$$\lim_{\substack{x\to 3^+\\x>3\\(x\neq 3)}} f(x)=\lim_{\substack{x\to 3^+\\(x>3)}}\frac{4\cancel{(x-3)}}{\cancel{(x-3)}}=\boxed{4}$$

$$\Rightarrow \lim_{x\to 3} f(x)= \text{d.n.e.}$$

Since

$$\lim_{x\to 3^-} f(x)=-4\neq 4=\lim_{x\to 3^+} f(x)$$

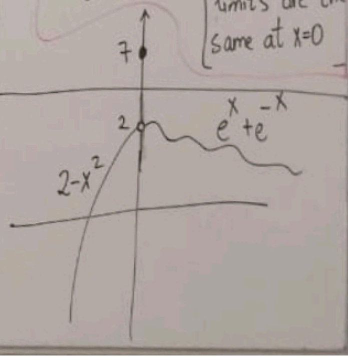


Ex.5.3:
$$f(x)=\begin{cases} 2-x^2, & x<0 \\ 7, & x=0 \\ e^x+e^{-x}, & x>0 \end{cases}$$

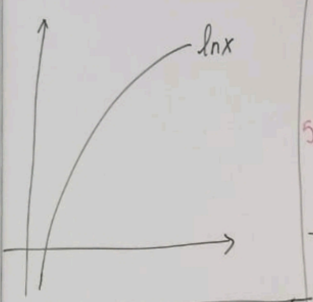
$$\lim_{\substack{x\to 0^-\\(x<0)}} f(x)=\lim_{x\to 0^-}(2-x^2)=2-0=\boxed{2}$$

$$\lim_{\substack{x\to 0^+\\(x>0)}} f(x)=\lim_{x\to 0^+}(e^x+e^{-x})=1+1=\boxed{2}$$

$$\Rightarrow \lim_{x\to 0} f(x)=2$$ [since right & left limits are the same at $x=0$



Ex 5-4: $\lim_{x\to0^+}(\ln x)=-\infty$



Ex 5.6: $a=\sqrt{a}\sqrt{a}$

$$\lim_{\substack{x\to8^+\\(x>8)}}\left(\frac{x^2-10x+16}{\sqrt{x-8}}\right)=\frac{0}{0}$$

$$=\lim_{x\to8^+}\frac{\overbrace{(x-8)}^{\sqrt{x-8}\,\cancel{\sqrt{x-8}}}(x-2)}{\cancel{\sqrt{x-8}}}=\frac{0}{0}$$

$$=\lim_{x\to8^+}(\sqrt{x-8})(x-2)=0(8-2)=\boxed{0}$$

p.42 Evaluate the following limits (if any):

5.5) $$\lim_{\substack{x\to7^-\\(x<7)}}\left(\frac{|x-7|}{x-7}\right)=\lim_{x\to7^-}\frac{-(x-7)}{(x-7)}=-1$$

5.6) $$\lim_{\substack{x\to7^+\\(x>7)}}\left(\frac{|x-7|}{x-7}\right)=\lim_{x\to7^+}\frac{(x-7)}{(x-7)}=1$$

$$\Rightarrow\ \lim_{x\to7}\frac{|x-7|}{x-7}=\text{d.n.e.}$$

5.8) $$\lim_{x\to0^+}\left(\frac{\sqrt{16+3x}-4}{x}\right)=\frac{0}{0}$$

$$=\lim_{x\to0^+}\left[\frac{(\sqrt{16+3x}-4)(\sqrt{16+3x}+4)}{x(\sqrt{16+3x}+4)}\right]$$

$$=\lim_{x\to0^+}\frac{\cancel{16}+3\cancel{x}-\cancel{16}}{\cancel{x}(\sqrt{16+3x}+4)}=\frac{3}{8}$$

$$|x|=\begin{cases}x, & x>0\\ -x, & x<0\end{cases}$$

5.11) $$\lim_{\substack{x\to0^+\\(x>0)}}\left(\frac{2x^2+3x|x|}{x|x|}\right)=$$

$$=\lim_{x\to0^+}\frac{2x^2+3x(x)}{x(x)}=\lim_{x\to0^+}\frac{5\cancel{x^2}}{\cancel{x^2}}=\boxed{5}$$

5.12) $$\lim_{\substack{x\to0^-\\(x<0)}}\left(\frac{2x^2+3x|x|}{x|x|}\right)=$$

$$=\lim_{x\to0^-}\frac{2x^2+3x(-x)}{x(-x)}$$

$$=\lim_{x\to0^-}\left(\frac{-x^2}{-x^2}\right)=1$$

HW:

p.37 → Ex. 5.1

p.39 → Ex. 5.5