Ch.4: Limits:

Ex. 4.3: Evaluate the following limit (if it exists):

$$\lim_{x\to -5}\left(\frac{x^2+6x+5}{x^2+7x+10}\right)=\frac{(-5)^2+6(-5)+5}{(-5)^2+7(-5)+10}=\frac{30-30}{35-35}=\left(\frac{0}{0}\right)$$

$\downarrow$ indeterminate form

$$=\lim_{\substack{x\to -5\\ x\neq -5}}\left[\frac{\cancel{(x+5)}(x+1)}{\cancel{(x+5)}(x+2)}\right]=\frac{-5+1}{-5+2}=\frac{-4}{-3}=\left(\frac{4}{3}\right)$$

Ex. 4.4: Evaluate the following limit (if it exists):

$$\lim_{x\to 0}\left(\frac{x^3-64}{x-4}\right)=\frac{-64}{-4}=\boxed{16}$$

Ex. 4.5: $\lim_{x\to 4}\left(\frac{x^3-64}{x-4}\right)=\frac{0}{0}$

$$=\lim_{\substack{x\to 4\\ x\neq 4}}\frac{\cancel{(x-4)}(x^2+4x+16)}{\cancel{(x-4)}}=(48)$$

Ex. 4.6: $\lim_{x\to 5}\frac{1}{|x-5|}=\frac{1}{0}=$ undefined (∞)

$\to 5 \leftarrow$

Ex. 4.7: (HW)

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Ex. 4.8: $\lim_{x\to 49}\left(\frac{\sqrt{x}-7}{x-49}\right)=\frac{0}{0}$

$$x-49=(\sqrt{x})^2-(7)^2=(\sqrt{x}-7)(\sqrt{x}+7)$$

$$=\lim_{\substack{x\to 49\\ (x\neq 49)}}\frac{\cancel{(\sqrt{x}-7)}}{\cancel{(\sqrt{x}-7)}(\sqrt{x}+7)}=\frac{1}{7+7}=\left(\frac{1}{14}\right)$$

Ex. 4.9: $\lim_{x\to 2}\left(\frac{x^3-7x+6}{x^2-5x+6}\right)=\left(\frac{0}{0}\right)$

$$\begin{array}{l|l} x^3-7x+6 & x-2 \\ \hline -x^3+2x^2 & x^2+2x-3 \\ \quad 2x^2-7x & \\ \quad -2x^2+4x & \\ \qquad -3x+6 & \\ \qquad\ 0\quad 0 & \end{array}$$

$$=\lim_{\substack{x\to 2\\ (x\neq 2)}}\frac{\cancel{(x-2)}(x^2+2x-3)}{\cancel{(x-2)}(x-3)}=\frac{4+4-3}{2-3}=\frac{5}{-1}=(-5)$$

$$\boxed{a^3-b^3=(a-b)(a^2+ab+b^2)}$$

Ex.4-10: $\lim_{x\to8}\left(\frac{2-\sqrt[3]{x}}{8-x}\right)=\frac{0}{0}$

$8-x=(2)^3-(\sqrt[3]{x})^3$

$=(\underset{\substack{\downarrow\\ a}}{2}-\underset{\substack{\downarrow\\ b}}{\sqrt[3]{x}})(4+2\sqrt[3]{x}+(\sqrt[3]{x})^2)$

$=\lim_{x\to8}\left[\frac{\cancel{(2-\sqrt[3]{x})}}{\cancel{(2-\sqrt[3]{x})}(4+2\sqrt[3]{x}+(\sqrt[3]{x})^2)}\right]$

$=\frac{1}{4+4+4}=\boxed{\frac{1}{12}}$

$a-b=(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})$

Ex.4-11: $\lim_{x\to3}\left(\frac{x^2-9}{x^2-6x+9}\right)=\frac{0}{0}$

$=\lim_{\substack{x\to3\\ x\neq3}}\left[\frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}(x-3)}\right]=\frac{6}{0}=$ d.n.e. (undefined)

$\hookrightarrow(\infty)$

Ex.4-12: $\lim_{x\to0}\left(\frac{\sqrt{9+12x}-3}{x}\right)=\frac{0}{0}$

$=\lim_{x\to0}\left[\frac{(\sqrt{9+12x}-3)(\sqrt{9+12x}+3)}{(\sqrt{9+12x}+3)(x)}\right]$

$=\lim_{\substack{x\to0\\ x\neq0}}\frac{\cancel{9}+12\cancel{x}-\cancel{9}}{(\sqrt{9+12x}+3)\cancel{(x)}}=\frac{12}{6}=2$

p.33 4-10: $\lim_{x\to7}\frac{\sqrt{4x+8}-6}{x-7}=\frac{0}{0}$

$=\lim_{x\to7}\frac{(\sqrt{4x+8}-6)(\sqrt{4x+8}+6)}{(x-7)(\sqrt{4x+8}+6)}=\lim_{\substack{x\to7\\ x\neq7}}\left[\frac{\overbrace{4x+8-36}^{4\cancel{(x-7)}}}{\cancel{(x-7)}(\sqrt{4x+8}+6)}\right]=\frac{4}{12}=\frac{1}{3}$

4.9) $\lim_{x\to0}\left(\frac{\sqrt{2x+1}-3}{x}\right)=\frac{-2}{0}=-\infty$

4.7) $\lim_{x\to3}\frac{x-3}{\sqrt{x+6}-3}=\frac{0}{0}$

$=\lim_{\substack{x\to3\\ x\neq3}}\left[\frac{\cancel{(x-3)}(\sqrt{x+6}+3)}{\underbrace{(\sqrt{x+6}-3)(\sqrt{x+6}+3)}_{\underbrace{x+6-9}_{\cancel{x-3}}}}\right]=3+3=6$

$c \to$ constant, $p>0$

(*) $\boxed{\lim_{x\to\mp\infty}\left(\frac{c}{x^p}\right)=0}$

$\frac{p(x)}{q(x)}$ > polynomials

$$\lim_{x\to\mp\infty}\left(\frac{p(x)}{q(x)}\right)=\begin{cases}\mp\infty, & \deg p(x)>\deg q(x)\\ \text{ratio of leading term coeffs.} & \text{if } \deg p(x)=\deg q(x)\\ 0, & \deg p(x)<\deg q(x)\end{cases}$$

p.33/4.11

$$\lim_{x\to\infty}\left(\frac{x(x^2-5x+14)}{7-4x^3}\right)=\lim_{x\to\infty}\left(\frac{x^3-5x^2+14x}{7-4x^3}\right)=$$

$$=\lim_{x\to\infty}\frac{\cancel{x^3}\left(1-\underset{\to 0}{\frac{5}{x}}+\underset{\to 0}{\frac{14}{x^2}}\right)}{\cancel{x^3}\left(\underset{0}{\frac{7}{x^3}}-4\right)}=\frac{1-0+0}{0-4}=\boxed{-\frac{1}{4}}$$

4.12)

$$\lim_{x\to\infty}\left[\frac{3x^2+12x+9}{(x^2-1)(x^2+1)}\right]=\lim_{x\to\infty}\left(\frac{3x^2+12x+9}{x^4-1}\right)=\lim_{x\to\infty}\frac{\cancel{x^2}\left(3+\underset{\to 0}{\frac{12}{x}}+\underset{\to 0}{\frac{9}{x^2}}\right)}{\underset{x^2}{\cancel{x^4}}\left(1-\underset{0}{\frac{1}{x^4}}\right)}$$

$$=\left(\lim_{x\to\infty}\frac{1}{x^2}\right)\cdot\left(\frac{3+0+0}{1-0}\right)=(0)(3)=\boxed{0}$$

$\frac{1}{\infty}=0$

4.13)

$$\lim_{x\to\infty}\left(\frac{2+3x-4x^4}{\sqrt{x}\,(1-17x+8x^3)}\right)=\lim_{x\to\infty}\left(\frac{2+3x-4x^4}{x^{1/2}-17x^{3/2}+8x^{7/2}}\right)$$

$$=\lim_{x\to\infty}\frac{\overset{\sqrt{x}}{x^4}\left(\underset{\to 0}{\frac{2}{x^4}}+\underset{\to 0}{\frac{3}{x^3}}-4\right)}{\cancel{x^{7/2}}\left(\underset{0}{\frac{1}{x^3}}-\underset{0}{\frac{17}{x^2}}+8\right)}=\left(\lim_{x\to\infty}\sqrt{x}\right)\left(\frac{0+0-4}{0-0+8}\right)=(\infty)\left(-\frac{1}{2}\right)$$

$$=-\infty$$