Parabolas:

Quadratic eqns.:

Solutions to the quadratic eqn.:

(A) $\boxed{ax^2+bx+c=0}$ $(a\neq 0)$

can be found using the quadratic formula:

(*) $$\boxed{x_{1,2}=\frac{-b\mp\sqrt{b^2-4ac}}{2a}}$$

Here we let $\boxed{\Delta=b^2-4ac}$ → discriminant, and the solns.

$x_{1,2}$ (if any) given by (*) are determined according to the sign of Δ as follows:

i) $\boxed{\Delta>0} \Rightarrow x_1, x_2$ are 2 real, distinct roots (solutions) of the given quadratic eqn. (A)

ii) $\boxed{\Delta=0} \Rightarrow \boxed{x_1=x_2=\frac{-b}{2a}}$; 2 real, identical solns. of eqn. (A)

iii) $\boxed{\Delta<0} \Rightarrow$ no real solutions of the eqn. (A)

* In eqn. (A),

i) if $a>0 \Rightarrow \cup$

ii) if $a<0 \Rightarrow \cap$

Ex. 2.1: Solve the equation

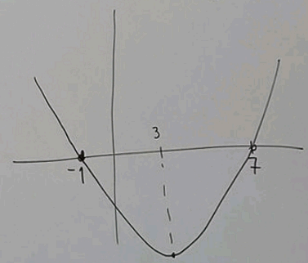
$$\boxed{x^2-6x-7=0}$$

$a=1>0 \Rightarrow \cup$

$$\left.\begin{array}{l} x^2-6x-7=0 \\ (x-7)(x+1)=0 \end{array}\right\} \boxed{x=7,\ x=-1}$$

x-intercepts of the parabola

Soln.: $\{7,-1\}$



Ex. 2.2: Solve $8x^2-6x-5=0$

$a=8>0 \Rightarrow \cup \qquad (b=-6,\ c=-5)$

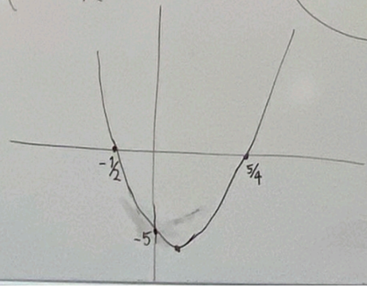
$$x_{1,2}=\frac{-b\mp\sqrt{b^2-4ac}}{2a}=\frac{-(-6)\mp\sqrt{(-6)^2-4(8)(-5)}}{2(8)}$$

$$=\frac{6\mp\sqrt{36+160}}{16}=\frac{6\mp 14}{16}=\begin{cases}\frac{6+14}{16}=\frac{\cancel{20}^{5}}{\cancel{16}_{4}}=\boxed{\frac{5}{4}}\\[4pt] \frac{6-14}{16}=\frac{-8}{16}=\boxed{\frac{-1}{2}}\end{cases}$$

$\Rightarrow$ Solns: $\left\{-\frac{1}{2}, \frac{5}{4}\right\}$

$8x^2-6x-5=0$

$(4x-5)(2x+1)=0 \Rightarrow x=\frac{5}{4},\ x=\frac{-1}{2}$

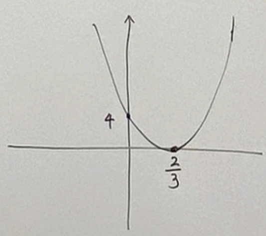


Ex. 2.3: Solve $9x^2-12x+4=0$

$a=9>0 \Rightarrow \cup$

$\Delta=b^2-4ac=(-12)^2-4(9)(4)$
$=144-144=0$

$$x_{1,2}=\frac{-b}{2a}=\frac{-(-12)}{2(9)}=\frac{\cancel{12}^{2}}{\cancel{18}_{3}}=\frac{2}{3}$$



$\Rightarrow$ Soln.: $\left\{\frac{2}{3}\right\}$ (2 real identical solns.)

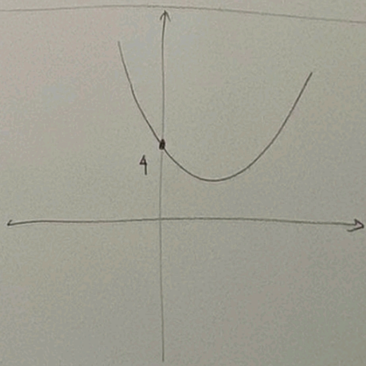
Ex. 2.4: Solve $3x^2+6x+4=0$

$a=3>0 \Rightarrow \cup$

$\Delta=b^2-4ac=(6)^2-4(3)(4)=36-48<0$

$\Rightarrow$ no real solns. of the given quadratic eqn.

$\Rightarrow$ Soln: $\{\varnothing\}$



Quadratic functions:

Ⓑ $\underbrace{f(x)=ax^2+bx+c}_{\text{quadratic function}}$ $(a\neq 0)$

$\Rightarrow$ graphs of such functions are parabolas.

* The vertex of the parabola is either the minimum or maximum point.

* The pt(s). where the graph of Ⓑ intersects the x-axis is(are) called the $\boxed{\text{x-intercepts.}}$

* The pt. for which $x=0$ is called the $\boxed{\text{y-intercept}}$ of the parabola.

Vertex of the parabola:

$$\boxed{f(x)=ax^2+bx+c}$$

$$= a\left(x^2+\frac{b}{a}x+\frac{c}{a}\right)$$

$$= a\left[\left(x+\frac{b}{2a}\right)^2-\frac{b^2}{4a^2}+\frac{c}{a}\right]$$

$$\left(x^2+\frac{b}{a}x+\frac{b^2}{4a^2}\right)$$

$$\boxed{f(x)=a\left(x+\frac{b}{2a}\right)^2+\left(c-\frac{b^2}{4a}\right)}$$

$$\underbrace{\text{Vertex: } \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)}$$

$a>0 \Rightarrow$ Vertex min. pt. of $\cup$

$a<0 \Rightarrow$ // max. pt. // $\cap$

Ex 2-5: Sketch the graph of

$$\boxed{f(x)=x^2-10x+16}$$

intercepts:

y intercept: $x=0 \Rightarrow f(0)=16 \Rightarrow (0,16)$: y-intercept

x-intercept(s): $(x-8)(x-2)=0$

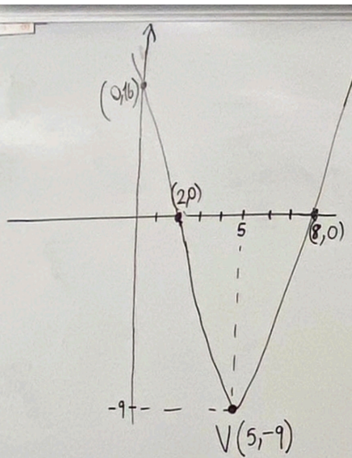
$(2,0), (8,0) \Rightarrow$ x-intercepts.

$f(x)=x^2-10x+16$, $a=1>0 \Rightarrow \cup$

Vertex: $-\frac{b}{2a}=-\frac{(-10)}{2(1)}=\frac{10}{2}=\boxed{5}$

$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)=(5, f(5))=\boxed{(5,-9)}$ Vertex

$f(5)=(5)^2-10(5)+16=25+16-50=\boxed{-9}$



Domain $f(x)$: $(-\infty,\infty)$

Range $f(x)$: $[-9,\infty)$

Find the vertex and x- and y-intercepts of the following parabolas. Sketch their graphs. Give the domain and range of the quadratic functions $f(x)$ determining the parabolas.

2-12) $f(x) = -x^2 + 12$

$a=-1<0 \Rightarrow \cap$; $b=0$, $c=12$

y-intercept: $x=0 \Rightarrow y=12$

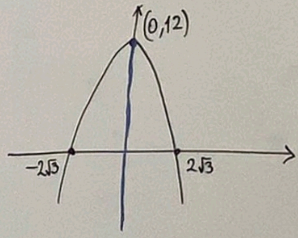
(0,12)

X-ints.: $-x^2+12=0 \Rightarrow x^2 = 12 = 3.4$

$\boxed{x = \mp 2\sqrt{3}}$

$\boxed{(2\sqrt{3},0), (-2\sqrt{3},0)}$

Vertex: $-\frac{b}{2a} = -\frac{0}{2(-1)} = 0 \Rightarrow f(0) = 12 \Rightarrow \boxed{(0,12) \to \text{Vertex}}$



Domain $f(x)$: $(-\infty,\infty)$ (or R)

Range $f(x)$: $\boxed{(-\infty, 12]}$

(not $[12,-\infty)$)!!!!

2-15) $y=\overbrace{x^2+10x+25}^{f(x)}$

$= (x+5)^2$

y-intercept: $\boxed{(0,25)}$

x-intercepts: $(x+5)^2=0$

$\Rightarrow x_1=x_2=-5$

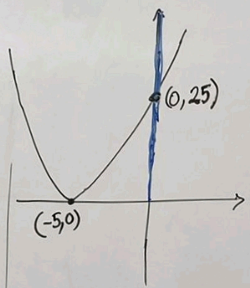
$\boxed{(-5,0): \text{double root}}$

$a=1>0 \Rightarrow \cup$

Vertex: $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

$-\frac{10}{2}=-5 \Rightarrow f(-5)=0$

$\Rightarrow$ Vertex: $(-5,0)$



Domain $f(x)$: $(-\infty,\infty)$

Range $f(x)$: $[0,\infty)$

2-20) $y=\underbrace{-3x^2+60x-450}_{f(x)}$

$a=-3<0 \Rightarrow \cap$

$f(x)=-3[\underbrace{x^2-20x+150}]=0$

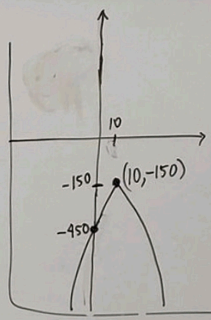
$\Delta=b^2-4ac=400-4(1)(150)<0$

$\Rightarrow$ no real roots of the quadratic

$\Rightarrow$ no x-intercepts! $\Rightarrow$ graph of the parabola does not intersect the x-axis.

*y-intercept: $(0,-450)$

Vertex: $\underbrace{\frac{-60}{2(-3)}}_{-\frac{b}{2a}}=10 \Rightarrow f(10)=-300+600-450=-150 \Rightarrow \boxed{V(10,-150)}$



Domain $f(x)$: $(-\infty,\infty)$

Range $f(x)$: $(-\infty,-150]$