ÇANKAYA UNIVERSITY

Department of Mathematics

MATH 113 - Mathematics for City Planners

2021-2022 Fall

FIRST MIDTERM EXAMINATION

16.11.2021, 17:30

-SOLUTIONS-

Question	Grade	Out of
1		16
2		18
3		14
4		21
5		28
6		9
Total		105

STUDENT NUMBER:

NAME-SURNAME:

SIGNATURE:

DURATION: 90 minutes

IMPORTANT NOTES:

1) Please make sure that you have written your student number and name above.

2) Check that the exam paper contains 6 problems.

3) SHOW ALL YOUR WORK. No points will be given to correct answers without reasonable work.

1. Find the solution sets of the following expressions:

(8 pts.) **a)** $\sqrt{2x+7} = x - 4$

$\left(\sqrt{2x+7}\right)^2 = (x-4)^2 \Rightarrow 2x+7 = x^2 - 8x + 16 \Rightarrow x^2 - 10x + 9 = 0$

$(x-9)(x-1) = 0 \rightarrow x=1, \; x=9$

$\Rightarrow$ $x=1$: $\sqrt{2(1)+7} = 3 \neq 1-4 = -3 \Rightarrow x=1$ doesn't satisfy the original eqn.

$x=9$: $\sqrt{2(9)+7} = 5 = 9-4 = 5$ ✓

$\boxed{\text{Soln.: } \{9\}}$

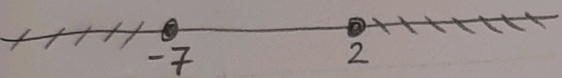
(8 pts.) **b)** $|2x+5| + 1 \geq 10 \Rightarrow |2x+5| \geq 10 - 1 = 9$

$\Rightarrow 2x+5 \geq 9$ or $2x+5 \leq -9$

$2x \geq 4$ or $2x \leq -14$

$x \geq 2$ or $x \leq -7$

$\boxed{\text{Soln.: } (-\infty, -7] \cup [2, \infty)}$



2. (10 pts.) **a)** Given $f(x) = \dfrac{1}{\sqrt{x-1}}$ and $g(x) = \log(3x-2)$; find the domain of $(f \circ g)(x) = ?$

$$(f \circ g)(x) = f(g(x)) = f(\log(3x-2)) = \frac{1}{\sqrt{\log(3x-2) - 1}}$$

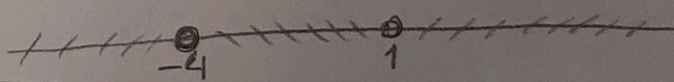
$\text{Dom}(f \circ g)(x) = \{x \mid \log(3x-2) > 1 = \log 10\} \Rightarrow 3x-2 > 10$

$3x > 12 \Rightarrow \boxed{x > 4}$

$\Rightarrow \boxed{\text{Soln.: } (4, \infty)}$

(8 pts.) **b)** Find the domain of $h(x) = e^{\left(\frac{x^2-x-2}{x^2+3x-4}\right)}$.

$\text{Dom } h(x) = \{x \mid \underset{(x+4)(x-1)}{x^2+3x-4} \neq 0\} = \{x \mid x \neq -4, \; x \neq 1\}$



$\boxed{\text{Soln.: } (-\infty, -4) \cup (-4, 1) \cup (1, \infty)}$

3. Consider the function $f(x) = -(x+1)^2 + 8x + 1$.

a) (6 points) Find the vertex, x-intercept(s) and y-intercept(s) of $f(x)$ (If any).

$f(x) = -(x+1)^2 + 8x + 1 = -(x^2+2x+1) + 8x + 1 = -x^2 + 6x$

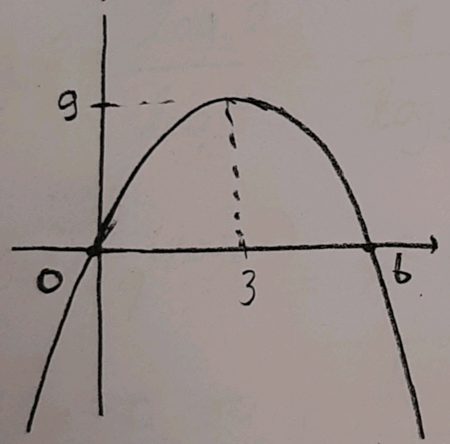
$a = -1,\ b = 6,\ c = 0.$

- Vertex: x-component $= \frac{-b}{2a} = 3.$ y-component: $y = f(3) = 9$ $\Big\}$ $\boxed{(3,9) \text{ is vertex}}$
- y-intercept: $x = 0 \Rightarrow y = f(0) = 0 \Rightarrow \boxed{(0,0) \text{ is y-int}}$
- x-intercept: $y = 0 \Rightarrow -x^2 + 6x = -x(x-6) = 0 \Rightarrow x = 0$ or $x = 6$

$\Rightarrow \boxed{(0,0) \text{ and } (6,0) \text{ are x-int.}}$

b) (4 points) Sketch the graph of f.

$a = -1 < 0 \Rightarrow \cap$



c) (4 points) Find the domain and the range of $f(x)$.

Using graph:

Domain $= \mathbb{R}$

Range $= (-\infty, 9]$

4) (7 points) a) Simplify the expression: $\ln \sqrt[5]{\frac{(x+2)^2(x+9)^3}{(x+1)^4}}$.

$$\ln\left[\frac{(x+2)^{2/5}\cdot(x+9)^{3/5}}{(x+1)^{4/5}}\right] = \ln(x+2)^{2/5} + \ln(x+9)^{3/5} - \ln(x+1)^{4/5}$$

$$= \boxed{\frac{2}{5}\ln(x+2) + \frac{3}{5}\ln(x+9) - \frac{4}{5}\ln(x+1)}$$

b) (7 points) Find the solution set of: $3^{(2\log_3 x)}9^{(\log_4 2)} = 6$.

$$3^{2\log_3 x} = 3^{\log_3 x^2} = x^2$$

$$\log_4 2 = \frac{\log_2 2}{\log_2 4} = \frac{1}{\log_2 2^2} = \frac{1}{2\log_2 2} = \frac{1}{2}$$

$$\Rightarrow 3^{(2\log_3 x)}\cdot 9^{(\log_4 2)} = x^2\cdot 9^{1/2} = 3x^2 = 6 \Rightarrow x^2 = 2 \Rightarrow x = \mp\sqrt{2}$$

But $x>0 \Rightarrow \boxed{\text{Solution set} = \{\sqrt{2}\}}$

c) (7 points) Find the solution set of: $\log_x(6-5x) = 2$.

$$\left.\begin{matrix} x>0 \\ x\neq 1 \end{matrix}\right\} \quad x^2 = 6-5x \Rightarrow x^2+5x-6=0 \Rightarrow (x+6)(x-1)=0$$

$$x=-6,\ x=1$$

But since x is base and $x>0, x\neq 1$, no valid x value solves the above eqn. $\Rightarrow \boxed{\text{Soln. set} = \{\emptyset\}}$

(No solution)

5. Do not use L'Hopital's rule !

(7 pts.) **a)** Evaluate $\lim_{x\to 2}\left(\frac{x^2-5x+6}{x^2-4}\right) = \frac{0}{0}$

$$= \lim_{x\to 2}\left(\frac{(x-3)\cancel{(x-2)}}{(x+2)\cancel{(x-2)}}\right) = \frac{2-3}{2+2} = \boxed{-\frac{1}{4}}$$

(7 pts.) **b)** Evaluate $\lim_{x\to 4}\left(\frac{x^2-2x-8}{\sqrt{x}-2}\right) = \frac{0}{0}$

$$= \lim_{x\to 4}\left(\frac{(x-4)(x+2)}{\sqrt{x}-2}\right) = \lim_{x\to 4}\frac{\cancel{(\sqrt{x}-2)}(\sqrt{x}+2)(x+2)}{\cancel{(\sqrt{x}-2)}}$$

$$= (2+2)(4+2) = \boxed{24}$$

(7 pts.) **c)** Evaluate $\lim_{x\to\infty}\frac{3-4x-2x^3}{5x^3-8x+1} = \lim_{x\to\infty}\frac{\cancel{x^3}\left(\frac{3}{x^3}-\frac{4}{x^2}-2\right)}{\cancel{x^3}\left(5-\frac{8}{x^2}+\frac{1}{x^3}\right)}$

$$= \frac{0-0-2}{5-0+0} = \boxed{-\frac{2}{5}}$$

(7 pts.) **d)** Evaluate $\lim_{x\to\infty}(\sqrt{x^2+2x+4}-x) = \infty - \infty$

$$= \lim_{x\to\infty}\frac{(\sqrt{x^2+2x+4}-x)(\sqrt{x^2+2x+4}+x)}{(\sqrt{x^2+2x+4}+x)} = \lim_{x\to\infty}\frac{\overbrace{\cancel{x^2}+2x+4-\cancel{x^2}}^{\cancel{x}\left(2+\frac{4}{x}\right)}}{\cancel{x}\left(\sqrt{1+\frac{2}{x}+\frac{4}{x^2}}+1\right)}$$

$$= \frac{2+0}{\sqrt{1+0+0}+1} = \frac{2}{2} = \boxed{1}$$

(8 pts.) **6.** For what values of the constants **a** and **b**, the function

$$f(x) = \begin{cases} \mathbf{a}x, & \text{if} \quad x < -1 \quad \text{or} \quad x > 3 \\ x^3 + \mathbf{b}, & \text{if} \quad -1 \le x \le 3 \end{cases}$$

is continuous for all x values ?

Since the functions "ax" and "x^3+b" are continuous everywhere, we should only check the jump points:

at $x=-1$: $\lim\limits_{\substack{x\to -1^- \\ (x<-1)}} f(x) = \lim\limits_{x\to -1^-} (ax) = \boxed{-a}$

$\lim\limits_{\substack{x\to -1^+ \\ (x>-1)}} f(x) = \lim\limits_{x\to -1^+} (x^3+b) = (-1)^3+b = \boxed{-1+b}$

For continuity at $x=-1$: $\boxed{-a=-1+b}$ $(a=1-b)$

at $x=3$: $\lim\limits_{\substack{x\to 3^- \\ (x<3)}} f(x) = \lim\limits_{x\to 3^-} (x^3+b) = (3)^3+b = \boxed{27+b}$

$\lim\limits_{\substack{x\to 3^+ \\ (x>3)}} f(x) = \lim\limits_{x\to 3^+} (ax) = \boxed{3a}$

For cont. at $x=3$: $\boxed{27+b=3a}$

$$\left.\begin{array}{l} a=1-b \\ 27+b=3a \end{array}\right\} 27+b=3(1-b) \Rightarrow 4b=-24 \Rightarrow \boxed{b=-6}$$

$$\Rightarrow a = 1-(-6) = \boxed{7}$$

Soln.: $\boxed{\begin{array}{l} a=7 \\ b=-6 \end{array}}$