# Fall 2020 - Math 113 - Midterm 1 Solutions

① Find the solution sets of the following expressions:

(a) $\log(6x^2-x-11)=0=\log 1$

$\Rightarrow 6x^2-x-11=1 \Rightarrow 6x^2-x-12=0 \Rightarrow (3x+4)(2x-3)=0$

$\Rightarrow x=-\frac{4}{3},\ x=\frac{3}{2}$

$\boxed{\text{Soln. set: } \left\{-\frac{4}{3}, \frac{3}{2}\right\}}$

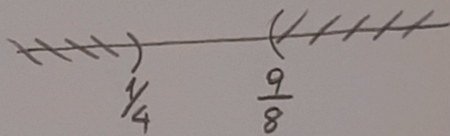
(b) $|3x-2|+x>\frac{5}{2} \Rightarrow |3x-2|>\frac{5}{2}-x$

$\Rightarrow 3x-2>\frac{5}{2}-x$ or $3x-2<-\left(\frac{5}{2}-x\right)$

$4x>\frac{5}{2}+2=\frac{9}{2}$ $\qquad$ $3x-x<-\frac{5}{2}+2$

$\boxed{x>\frac{9}{8}}$ or $2x<\frac{1}{2}$

$\boxed{x<\frac{1}{4}}$



$\boxed{\text{Soln. set: } \left(-\infty, \frac{1}{4}\right) \cup \left(\frac{9}{8}, \infty\right)}$

(c) $\sqrt{2x+7}=x-4$

Squaring both sides: $\left(\sqrt{2x+7}\right)=(x-4)^2 \Rightarrow 2x+7=x^2-8x+16$

$\Rightarrow x^2-10x+9=0 \Rightarrow (x-9)(x-1)=0 \Rightarrow \boxed{x=1 \text{ or } x=9}$

Check $x=1, x=9$ at the original eqn.:

$x=1$: $\sqrt{2(1)+7}=3 \neq 1-4=-3 \Rightarrow x=1$ doesn't satisfy the given eqn.

$x=9$: $\sqrt{2(9)+7}=5 \stackrel{\checkmark}{=} 9-4=5 \Rightarrow x=9$ satisfies the given eqn.

$\Rightarrow \boxed{\text{Soln. set: } \{9\}}$

② $f(x)=\dfrac{1}{\sqrt{x-1}}$ , $g(x)=\log(3x-2)$

$$\Rightarrow (f\circ g)(x)=f(g(x))=f(\log(3x-2))=\frac{1}{\sqrt{\log(3x-2)-1}}$$

$$\text{Domain } (f\circ g)(x)=\{x \mid \underbrace{\log(3x-2)>1}\} \quad (1=\log 10)$$

$$\Downarrow$$

$$\log(3x-2)>\log 10$$

$$\Rightarrow 3x-2>10 \Rightarrow 3x>12$$

$$\Rightarrow \boxed{x>4}$$

$$\therefore \boxed{\text{Domain } (f\circ g)(x): (4,\infty)}$$

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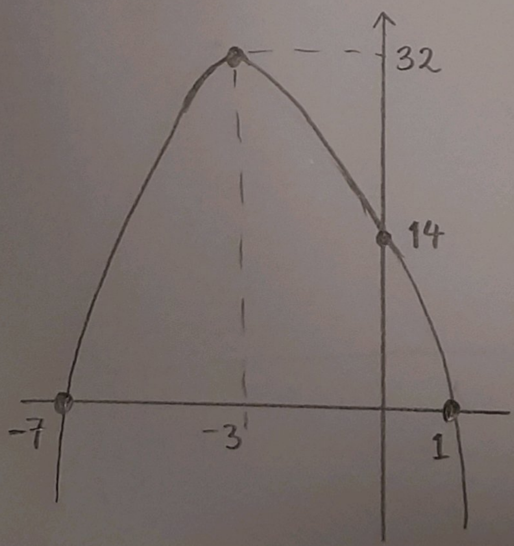
③ $y=-2(x+3)^2+32$:

Vertex: $\boxed{(-3,32)}$

y-intercept: $x=0 \Rightarrow y=-2(9)+32=14 \Rightarrow \boxed{(0,14)}$

x-intercept(s): $y=0 \Rightarrow -2(x+3)^2+32=0 \Rightarrow (x+3)^2=16$

$\boxed{(1,0) \,\&\, (-7,0)}$ $\quad \Rightarrow x+3=\mp 4 \Rightarrow x=-7$ or $x=1$




Range: $(-\infty,32]$

Vertex: $(-3,32)$

y-intercept: $(0,14)$

x-intercepts: $(1,0) \,\&\, (-7,0)$

④ line $\ell$ perpendicular to $\ell_1: 3x+2y-4=0$ and passing through $(3,1)$:

$\ell_1: 2y=-3x+4 \Rightarrow y=-\frac{3}{2}x+2 \Rightarrow \ell_1$ has slope $-\frac{3}{2}$

$\Rightarrow$ line $\ell$ perpendicular to $\ell_1$ has slope: $\boxed{m=\frac{2}{3}}$

$\ell$ with slope $m=\frac{2}{3}$ passing through $(3,1)$:

$$y-1=\left(\frac{2}{3}\right)(x-3)=\frac{2}{3}x-2 \Rightarrow y=\frac{2}{3}x-2+1$$

$$\Rightarrow \boxed{y=\frac{2}{3}x-1}$$

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⑤ $\log_x(6-4x-x^2)=2 \Rightarrow x^2=6-4x-x^2 \Rightarrow 2x^2+4x-6=0$

$(x>0, x\neq 1)$

$\Rightarrow 2(x^2+2x-3)=0$

$(x+3)(x-1)=0 \Rightarrow \boxed{x=-3 \text{ or } x=1}$

$\Rightarrow$ But $x=-3$ is not possible since $x>0$

$x=1$ " " " " $x\neq 1$

$\Rightarrow$ $\boxed{\text{Solution set: } \{\emptyset\}}$

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⑥ $\frac{1}{3}\ln x+3\ln(x^2)-2\ln(x-1)-3\ln(x-2)$

$=\ln x^{1/3}+\ln(x^2)^3-\ln(x-1)^2-\ln(x-2)^3$

$$=\ln\left[\frac{x^{1/3}\cdot x^6}{(x-1)^2(x-2)^3}\right]=\boxed{\ln\left[\frac{x^{19/3}}{(x-1)^2(x-2)^3}\right]}$$

## Bonus Questions

(A) $\log_3(x+5) = \boxed{\dfrac{\ln(x+5)}{\ln 3}}$ (by change-of-base formula)

(B) $3^{4x} = 9^{x+1} \Rightarrow 3^{4x} = ((3)^2)^{x+1} = 3^{2x+2}$

$\Rightarrow 4x = 2x+2 \Rightarrow 2x = 2 \Rightarrow \boxed{x=1}$