

Fall 2020, Math 113 - Final Exam

January 11, 2021; 10⁰⁰ - 11³⁰ (on Webonline)

Make sure; to submit your solutions as a single "pdf" file, scanned vertically, in batch mode. Please write firmly and clearly.

(20pts) ① Find the domain of the given function: (SHOW WORK!)

a- $f(x) = \sqrt{x^2 - 4x - 5}$

c- $g(x) = e^{\left(\frac{x^2 - x - 2}{x^2 + 3x - 4}\right)}$

b- $f(x) = \ln \left[\frac{(x+1)(x-2)}{(x+4)(x-5)} \right]$

(20pts) ② Evaluate the following limits (Do not use L'Hôpital's rule!)

a- $\lim_{x \rightarrow -1} \left(\frac{x^3 + x^2 + 3x + 3}{x^3 + 1} \right) = ?$

b- $\lim_{x \rightarrow 4} \left(\frac{x^3 - 2x^2 - 5x - 12}{\sqrt{x} - 2} \right) = ?$

c- $\lim_{x \rightarrow \infty} \left(\frac{5x^4 - 2x^3 + x}{3x^3 - 7x - x^4 + 2} \right) = ?$

d- Let $f(x) = \begin{cases} \frac{x^2 + 2x}{x^2 + 3x + 2} & \text{if } x < -2 \\ a & \text{if } x = -2 \\ \frac{x^2 + bx + c}{x^2 - 2x - 8} & \text{if } x > -2 \end{cases}$

Find the value of the constants a, b, c if $f(x)$ is continuous at $x = -2$.

(20 pts.) ③ Let $f(x) = \begin{cases} x^2 - 7x + 20 & \text{if } x < 2 \\ 10 & \text{if } x = 2 \\ -6\sqrt{2x} + 22 & \text{if } x > 2 \end{cases}$

Find $f'(2)$ if it exists. (Hint: use the definition

of the derivative: $f'(x) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$.

Do not use L'Hôpital's rule.)

(20 pts.) ④ a) Find $f'(x)$ if $f(x) = e^{x^3} + \ln(2x^2 + 3)$

b) Find y' if $x^3y + e^{xy^2} = 5$

(20 pts.) ⑤ Let $A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 2 & -1 \\ 4 & -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -2 & 2 \\ 1 & -1 & -2 \\ 0 & 3 & 1 \end{bmatrix}$

a) Find A^2

b) Find AB

c) Find $B^T A^T$

d) Find $3A - 2B$

(20 pts.) ⑥ Find the trace of the following matrices:

a) $\begin{bmatrix} 1 & 2 & 2 \\ 2 & -1 & 1 \\ 2 & 3 & -4 \end{bmatrix}$,

b) $\begin{bmatrix} 3 & -2 & 1 \\ 0 & -2 & 1 \\ 3 & 1 & 2 \end{bmatrix}^T$

(if possible!)

c) $\begin{pmatrix} \begin{bmatrix} 3 & -2 & 1 \\ 0 & -2 & 1 \\ 3 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -2 & 1 \\ 0 & -2 & 1 \end{bmatrix}^T \end{pmatrix}^T$,

d) $\begin{bmatrix} 3 & -2 & 1 \\ 0 & -2 & 1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 0 & -2 \\ 3 & 1 \end{bmatrix}$

(10 pts.) Bonus Question:

Find an equation of the tangent line
drawn to the graph of the function

$$f(x) = x^2 - 2x + 5$$

at $x=2$

GOOD

LUCK!