Examples: (11.1, 11.2, 11.4) Evaluate the derivatives of the following functions:

1.) $f(x) = 3 - 5x^2 \Rightarrow f'(x) = 0 - 10x = -10x$

2.) $f(x) = x^4 - \sqrt[3]{x} \Rightarrow f(x) = x^4 - x^{1/3} \Rightarrow f'(x) = 4x^3 - \frac{1}{3}x^{\frac{1}{3}-1}$

$= \boxed{4x^3 - \frac{1}{3}x^{-2/3}}$

3.) $y = 4x^4 + x^3 - \frac{9}{2}x^2 + 8x \Rightarrow y' = \boxed{16x^3 + 3x^2 - 9x + 8}$

4.) $y = 2 \cdot x^{-14/5} \Rightarrow y' = 2\left(-\frac{14}{5}\right)x^{-\frac{14}{5}-1} = \boxed{-\frac{28}{5}x^{-\frac{19}{5}}}$

5.) $f(x) = \frac{x^3}{3} - \frac{3}{x^3} = \frac{1}{3}x^3 - 3x^{-3} \Rightarrow f'(x) = \frac{1}{3}3x^2 - 3(-3)x^{-4} = \boxed{x^2 + \frac{9}{x^4}}$

6.) $f(x) = \frac{3}{\sqrt[4]{x^3}} \Rightarrow f(x) = 3x^{-3/4} \Rightarrow f'(x) = 3\left(-\frac{3}{4}\right)x^{-\frac{3}{4}-1} = \boxed{-\frac{9}{4}x^{-7/4}}$

7.) $y = -9x^{1/3} + 5x^{-2/5} \Rightarrow y' = -9\left(\frac{1}{3}\right)x^{-\frac{2}{3}} + 5\left(-\frac{2}{5}\right)x^{-7/5} = \boxed{-3x^{-2/3} - 2x^{-7/5}}$

8.) $f(x) = \frac{2x^2}{\sqrt{x}} = 2x^2 \cdot x^{-1/2} = 2x^{3/2} \Rightarrow f'(x) = \cancel{2}\left(\frac{3}{\cancel{2}}\right)x^{\frac{3}{2}-1} = \boxed{3x^{1/2}}$

9.) $f(x) = \frac{7x^3 + x}{6\sqrt{x}} = \frac{7x^3}{6\sqrt{x}} + \frac{x}{6\sqrt{x}} = \frac{7}{6}x^{3-\frac{1}{2}} + \frac{1}{6}x^{1-\frac{1}{2}} = \frac{7}{6}x^{5/2} + \frac{1}{6}x^{1/2}$

$\Rightarrow f'(x) = \frac{7}{6} \cdot \frac{5}{2}x^{\frac{5}{2}-1} + \frac{1}{6}\frac{1}{2}x^{-1/2} = \boxed{\frac{35}{12}x^{3/2} + \frac{1}{12}x^{-1/2}}$

10.) $y = x^2(x^3 - 2x) = x^5 - 2x^3 \Rightarrow y' = \boxed{5x^4 - 6x^2}$

11.) $y = (7x^3 + 14x^2 - 6)(x^8 - 18) \Rightarrow y' = \boxed{(21x^2 + 28x)(x^8 - 18) + (7x^3 + 14x^2 - 6)(8x^7)}$

12.) $f(x) = (x^2 + 4x^3)\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) \Rightarrow \boxed{f'(x) = (2x + 12x^2)\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) + (x^2 + 4x^3) \cdot \left(\frac{1}{2}x^{-1/2} + \frac{1}{2}x^{-3/2}\right)}$

13) $f(x)=\frac{(3x-1)(x+7)}{(x+5)}=\frac{3x^2+20x-7}{x+5}$ (use quotient rule)

$$\Rightarrow f'(x)=\frac{(6x+20)(x+5)-(1)(3x^2+20x-7)}{(x+5)^2}$$

$$=\frac{6x^2+50x+100-3x^2-20x+7}{(x+5)^2}=\boxed{\frac{3x^2+30x+107}{(x+5)^2}}$$

14) $y=1-\frac{5}{2x+5}+\frac{2x}{3x+1}\Rightarrow y'=0-\frac{0(2x+5)-(2)(5)}{(2x+5)^2}+\frac{(2)(3x+1)-(3)(2x)}{(3x+1)^2}$

$$\Rightarrow y'=\boxed{\frac{10}{(2x+5)^2}+\frac{2}{(3x+1)^2}}$$

15) $y=\frac{x-5}{(x+2)(x+4)}=\frac{x-5}{x^2+6x+8}\Rightarrow y'=\frac{(1)(x^2+6x+8)-(2x+6)(x-5)}{(x^2+6x+8)^2}$

$$\Rightarrow y'=\frac{x^2+6x+8-2x^2+10x-6x+30}{(x^2+6x+8)^2}=\boxed{\frac{-x^2+10x+38}{(x^2+6x+8)^2}}$$

16) $f(x)=\frac{(9x-1)(3x+2)}{(4-5x)}=\frac{27x^2+15x-2}{(4-5x)}\Rightarrow f'(x)=\frac{(54x+15)(4-5x)-(-5)(27x^2+15x-2)}{(4-5x)^2}$

$$\Rightarrow f'(x)=\boxed{\frac{-135x^2+216x+50}{(4-5x)^2}}$$

17) $f=\frac{7x^4-x^2}{(x-1)}\Rightarrow f'=\frac{(28x^3-2x)(x-1)-(1)(7x^4-x^2)}{(x-1)^2}$

18) $f(x)=(x^3+5x)(x^2-\sqrt{x})\Rightarrow f'(x)=(3x^2+5)(x^2-\sqrt{x})+(2x-\frac{1}{2}x^{-1/2})(x^3+5x)$

19) $y=(x^2+1)(3x+4)(x-8)\Rightarrow y'=(2x)[(3x+4)(x-8)]+[3(x-8)+1(3x+4)](x^2+1)$

$\Rightarrow y'=(2x)(3x^2-20x-32)+(x^2+1)(6x-20)=\boxed{12x^3-60x^2-58x-20}$

20) $y=(2x+3)(x^7-4x^2)(1+x+x^2)$

$y'=(2)[(x^7-4x^2)(1+x+x^2)]+(2x+3)[(7x^6-8x)(1+x+x^2)+(x^7-4x^2)(1+2x)]$

Derivatives of Logarithms and Exponentials:

① $f(x)=\ln x \Rightarrow f'(x)=\frac{1}{x}$	② $f(x)=\log_b x \Rightarrow f'(x)=\frac{1}{x}\cdot\frac{1}{\ln b}$ $\left(\log_b x=\frac{\ln x}{\ln b}\right)$ $(b>0, b\neq 1)$
③ $f(x)=e^x \Rightarrow f'(x)=e^x$	④ $f(x)=b^x \Rightarrow f'(x)=b^x\cdot\ln b$

Examples: Find the derivatives of the following functions:

1) $y=3e^x+x^3+\sqrt[5]{x^2}+4\ln x+5 \Rightarrow \boxed{y'=3e^x+3x^2+\frac{2}{5}x^{-3/5}+\frac{4}{x}}$

2) $y=(x^2+x+1)(x^3+e^x+\ln x)$

$y'=(2x+1)(x^3+e^x+\ln x)+(x^2+x+1)\left(3x^2+e^x+\frac{1}{x}\right)$

3) $f(x)=x^4\cdot 2^x \Rightarrow f'(x)=4x^3\cdot 2^x+x^4\cdot 2^x\cdot\ln 2$

4) $f(x)=\dfrac{3^x}{1+x^2} \Rightarrow f'(x)=\dfrac{(3^x\cdot\ln 3)(1+x^2)-(2x)(3^x)}{(1+x^2)^2}$

5) $y=(x^3+5)\cdot(\log_5 x) \Rightarrow y'=(3x^2)(\log_5 x)+(x^3+5)\cdot\dfrac{1}{x\cdot\ln 5}$

6) $y=\dfrac{\log_7 x+x^3}{x^2+x+8} \Rightarrow y'=\dfrac{\left(\frac{1}{x\ln 7}+3x^2\right)(x^2+x+8)-(2x+1)(\log_7 x+x^3)}{(x^2+x+8)^2}$

7) $y=\dfrac{\ln x}{e^x} \Rightarrow y'=\dfrac{\left(\frac{1}{x}\right)(e^x)-(e^x)\cdot(\ln x)}{(e^x)^2}=\dfrac{\cancel{e^x}\left(\frac{1}{x}-\ln x\right)}{e^{\cancel{2x}}}$

$\Rightarrow \boxed{y'=\dfrac{\frac{1}{x}-\ln x}{e^x}}$

8) $y=(\ln x)(\sqrt{x}) \Rightarrow y'=\frac{1}{x}\cdot\sqrt{x}+\left(\frac{1}{2}x^{-1/2}\right)(\ln x)=\frac{1}{\sqrt{x}}+\frac{1}{2\sqrt{x}}(\ln x)$

$\Rightarrow \boxed{y'=\frac{1}{\sqrt{x}}\left[1+\frac{\ln x}{2}\right]}$