ÇANKAYA UNIVERSITY
Department of Mathematics

MATH 113 - Mathematics for City Planners

2019-2020 Fall

FIRST MIDTERM EXAMINATION
05.11.2019, 17:30

SOLUTIONS

Question	Grade	Out of
1		16
2		15
3		18
4		15
5		19
6		22
Total		105

STUDENT NUMBER:

NAME-SURNAME:

SIGNATURE:

INSTRUCTOR: B. K.

DURATION: 90 minutes

IMPORTANT NOTES:

1) Please make sure that you have written your student number and name above.

2) Check that the exam paper contains 6 problems.

3) Show all your work. No points will be given to correct answers without reasonable work.

1. Find the solution sets of the following expressions. Clearly indicate the solution sets.

a) (8 points) $\sqrt{x+3}+1 = 3\sqrt{x}$

$$\left(\sqrt{x+3}+1\right)^2 = \left(3\sqrt{x}\right)^2 \Rightarrow x+3+2\sqrt{x+3}+1 = 9x$$

$$\Rightarrow 2\sqrt{x+3} = 8x-4 \Rightarrow \sqrt{x+3} = 4x-2 \Rightarrow x+3 = (4x-2)^2 = 16x^2-16x+4$$

$$\Rightarrow 16x^2-17x+1=0 \Rightarrow (16x-1)(x-1)=0 \Rightarrow \boxed{x=\frac{1}{16} \text{ or } x=1}$$

Check these in the original eqn.:

$x=1$: $\sqrt{1+3}+1 = 2+1=3=3\sqrt{1}$ ✓

$x=\frac{1}{16}$: $\sqrt{\frac{1}{16}+3}+1 = \frac{7}{4}+1 = \frac{11}{4} \neq 3\sqrt{\frac{1}{16}} = \frac{3}{4}$ X

So the only soln. is $x=1$ $\Rightarrow \boxed{\text{Soln. set} = \{1\}}$

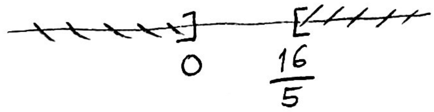
b) (8 points) $\left|\frac{3x-8}{2}\right| + x \geq 4$

$$\left|\frac{3x-8}{2}\right| \geq 4-x \Rightarrow \frac{3x-8}{2} \geq 4-x \quad \text{or} \quad \frac{3x-8}{2} \leq -(4-x)$$

$$3x-8 \geq 8-2x \qquad 3x-8 \leq -8+2x$$

$$5x \geq 16 \qquad x \leq 0$$

$$x \geq \frac{16}{5}$$



$$\boxed{\text{Soln. set} = (-\infty, 0] \cup [\tfrac{16}{5}, \infty)}$$

2. Consider the function $f(x) = -(x+1)^2 + 8x + 1$.

a) (6 points) Find the vertex, x-intercept(s) and y-intercept(s) of $f(x)$ (If any).

$$f(x) = -(x+1)^2 + 8x + 1 = -(x^2+2x+1) + 8x + 1 = -x^2 + 6x$$

$(\cap)$ $a=-1,\ b=6,\ c=0$

$-\frac{b}{2a} = \frac{-6}{-2} = 3$

Vertex: $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right) = \boxed{(3,9)}$

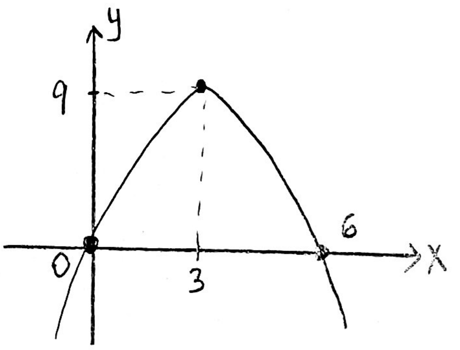
$f(3) = -9 + 18 = 9$

y-intercept: $x=0 \Rightarrow y=0 \Rightarrow \boxed{(0,0)}$

x-intercept(s): $y=0 \Rightarrow -x^2+6x = -x(x-6) = 0 \Rightarrow x=0,\ x=6$

$\boxed{(0,0),(6,0)}$

b) (5 points) Sketch the graph of f.



c) (4 points) Find the domain and the range of $f(x)$.

Domain: $\mathbb{R}$

Range: $(-\infty, 9]$

(using the graph above)

3. Consider the functions $f(x) = 2^{x+1}$ and $g(x) = \sqrt{3x^2 - 12}$.

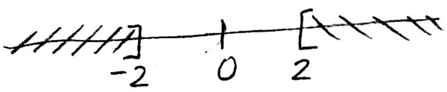
a) (3 points) Find the domain of the function $f(x)$.

$f(x) = 2^{x+1} \Rightarrow \boxed{\mathrm{Dom} f = \mathbb{R}}$

b) (6 points) Find the domain of the function $g(x)$.

$\mathrm{Dom}\, g = \{x \mid 3x^2 - 12 \geq 0\} = \boxed{(-\infty, -2] \cup [2, \infty)}$

$3x^2 \geq 12 \Rightarrow x^2 \geq 4 \Rightarrow x \geq 2$ or $x \leq -2$



c) (4 points) Find the function $(g \circ f)(x)$.

$(g \circ f)(x) = g(f(x)) = g(2^{x+1}) = \sqrt{3(2^{x+1})^2 - 12}$

$= \sqrt{3.2^{2x+2} - 12} = \sqrt{3.2^{2x}.2^2 - 12} = \sqrt{12(2^x - 1)}$

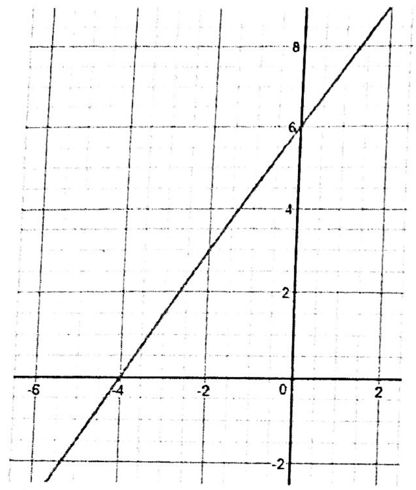
$= \boxed{2\sqrt{3(2^x - 1)}}$

d) (5 points) Find the domain of the function $(g \circ f)(x)$.

$\mathrm{Dom}(g \circ f)(x) = \{x \mid 3(2^{2x} - 1) \geq 0\} = \boxed{[0, \infty)}$

$2^{2x} \geq 1 = 2^0 \Rightarrow 2x \geq 0 \Rightarrow x \geq 0$

4. a) (5 points) Find the equation of the following line l_1 :



l_1 passing through $(0,6)$ & $(-4,0)$ $\Rightarrow$

$$\text{slope} = m_1 = \frac{6-0}{0-(-4)} = \frac{6}{4} = \frac{3}{2}$$

$$\text{eqn. of } l_1: \; y-0 = \frac{3}{2}(x-(-4)) = \frac{3}{2}x+6$$

$$\Rightarrow \boxed{y = \frac{3}{2}x+6}$$

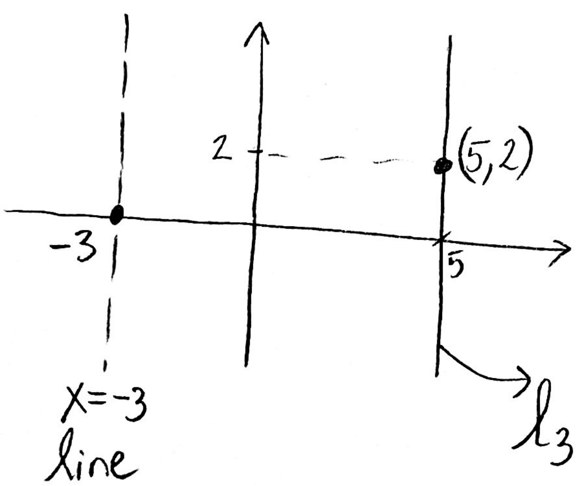
b) (6 points) Find the equation of the line l_2 passing through the point $(6, 2)$ and perpendicular to the line l_1 given in part a).

l_2 with slope $m_2 = \frac{-1}{m_1} = -\frac{2}{3}$

$$\text{eqn. of } l_2: \; y-2 = \left(-\frac{2}{3}\right)(x-6) \Rightarrow y = -\frac{2}{3}x+4+2$$

$$\boxed{y = -\frac{2}{3}x+6}$$

c) (4 points) Find the equation of the line l_3 passing through the point $(5, 2)$ and parallel to the line $x = -3$.



eqn. of l_3: $\boxed{x=5}$

5. (6 points) a) Simplify the expression as much as possible: $\ln \sqrt[5]{\frac{(x+2)^2(x+9)^3}{(x+1)^4}}$.

$$\ln\left[\frac{(x+2)^{2/5}\cdot(x+9)^{3/5}}{(x+1)^{4/5}}\right] = \ln(x+2)^{2/5} + \ln(x+9)^{3/5} - \ln(x+1)^{4/5}$$

$$= \boxed{\frac{2}{5}\ln(x+2) + \frac{3}{5}\ln(x+9) - \frac{4}{5}\ln(x+1)}$$

b) (6 points) Find the solution set of: $3^{(2\log_3 x)}9^{(\log_4 2)} = 6$.

$$3^{2\log_3 x} = 3^{\log_3 x^2} = x^2$$

$$\log_4 2 = \frac{\log_2 2}{\log_2 4} = \frac{1}{\log_2 2^2} = \frac{1}{2\log_2 2} = \frac{1}{2}$$

$$\Rightarrow 3^{(2\log_3 x)}\cdot 9^{(\log_4 2)} = x^2\cdot 9^{1/2} = 3x^2 = 6 \Rightarrow x^2 = 2 \Rightarrow x = \mp\sqrt{2}$$

But $\log_3 x$ is defined only for $x>0 \Rightarrow \boxed{\text{Soln.set} = \{\sqrt{2}\}}$

c) (7 points) Find the solution set of: $\log_x(6-5x) = 2$.

$\left.\begin{matrix} x>0 \\ x\neq 1\end{matrix}\right\}$ $\log_x(6-5x) = 2 \Rightarrow x^2 = 6-5x \Rightarrow (x+6)(x-1) = 0$

$x^2+5x-6=0$ $\quad \boxed{x=-6 \text{ or } x=1}$

(x is base)

But since x is base and $x>0, x\neq 1$, no valid x value solves the above eqn. $\Rightarrow$ $\boxed{\text{Soln.set} = \{\emptyset\} \text{ (No solution)}}$

6. Evaluate the following limits (if they exist): (SHOW YOUR WORK!)

(5 pts.) **a)** $\lim_{x\to 2^+}\left(2-\frac{1}{x-2}\right) = 2-\frac{1}{0^+} = \boxed{-\infty}$ (D.N.E.)

$x \to 2^+$

$x > 2$

$x - 2 > 0$

(6 pts.)**b)** $\lim_{x\to\infty}\left(\frac{3}{x}-\frac{2x^2}{1+x^2}\right) = \frac{3}{\infty} - \lim_{x\to\infty}\left[\frac{2\cancel{x^2}}{\cancel{x^2}\left(\frac{1}{x^2}+1\right)}\right]$

$$= 0 - \left(\frac{2}{0+1}\right) = \boxed{-2}$$

(6 pts.)**c)** $\lim_{x\to 4}\frac{x-4}{4-\sqrt{x+12}} = \left(\frac{0}{0}\right)$

$$= \lim_{x\to 4}\frac{(x-4)(4+\sqrt{x+12})}{(4-\sqrt{x+12})(4+\sqrt{x+12})} = \lim_{x\to 4}\frac{\cancel{(x-4)}(4+\sqrt{x+12})}{\underbrace{(16-(x+12))}_{-\cancel{(4-x)}}}$$

$$= -(4+\sqrt{4+12}) = -(4+4) = \boxed{-8}$$

(5 pts.)**d)** $\lim_{x\to\infty}\frac{3-4x-2x^3}{5x^3-8x+1} = \lim_{x\to\infty}\left\{\frac{\cancel{x^3}\left[\frac{3}{x^3}-\frac{4}{x^2}-2\right]}{\cancel{x^3}\left[5-\frac{8}{x^2}+\frac{1}{x^3}\right]}\right\} = \frac{0-0-2}{5-0+0} = -\frac{2}{5}$