ÇANKAYA UNIVERSITY

Department of Mathematics

MATH 113-Mathematics for City Planners

FIRST MIDTERM EXAMINATION

22.11.2016

SOLUTIONS

Question	Grade	Out of
1		15
2		15
3		20
4		16
5		16
6		18
Total		100

STUDENT NUMBER:

NAME-SURNAME:

SIGNATURE:

INSTRUCTOR:

DURATION: 60 minutes

IMPORTANT NOTES:

1) Please make sure that you have written your student number and name above.

2) Check that the exam paper contains 6 problems.

3) Show all your work. No points will be given to correct answers without reasonable work.

4) Your exam will not be graded if you don't take the exam at the right place.

1. Let ℓ be the line passing through the point $(4, -2)$ that is parallel to the line $\ell_1 : 2x - 5y + 3 = 0$.

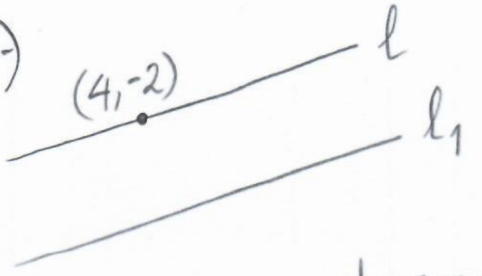
   a. Write an equation of ℓ.

   b. Find x-intercept and y-intercept of ℓ.

   c. Draw the graph of ℓ.

$\ell_1: 5y = 2x + 3 \Rightarrow y = \frac{2}{5}x + \frac{3}{5}$

a-)



$\ell // \ell_1$

$m = \text{slope of } \ell = \text{slope of } \ell_1 = m_1 = \frac{2}{5}$

$\Rightarrow$ eqn. of ℓ: $\left.\begin{array}{l} \text{slope} = \frac{2}{5} \\ \text{pt. } (4,-2) \end{array}\right\}$ $y - (-2) = \frac{2}{5}(x - 4)$

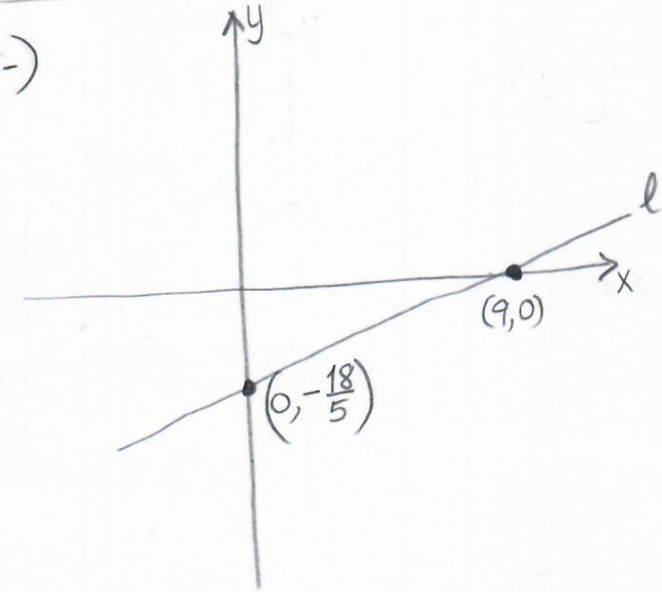
$y = \frac{2}{5}x - \frac{8}{5} - 2 = \frac{2}{5}x - \frac{18}{5}$

$\boxed{y = \frac{2}{5}x - \frac{18}{5}}$

b-) $x = 0 \Rightarrow y = -\frac{18}{5} \Rightarrow \left(0, -\frac{18}{5}\right) \rightarrow$ y-intercept of ℓ

$y = 0 \Rightarrow x = 9 \Rightarrow (9, 0) \rightarrow$ x-intercept of ℓ

c-)



$\ell: \boxed{y = \frac{2}{5}x - \frac{18}{5}}$

2. Let ℓ be the line passing through the point $(3, 1)$ that is perpendicular to the line $\ell_3 : x - 3y + 5 = 0$

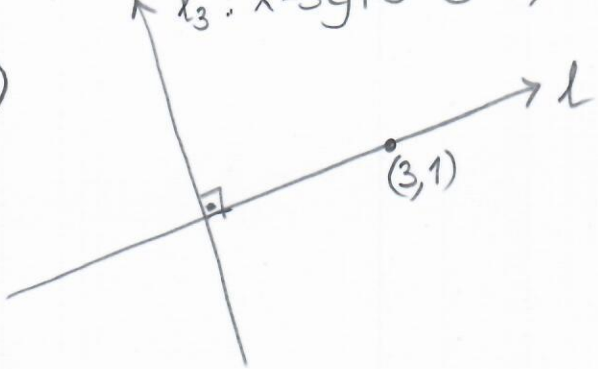
a. Write an equation of ℓ.

b. Find x-intercept and y-intercept of ℓ.

c. Draw the graph of ℓ.

$\ell_3 : x - 3y + 5 = 0 \Rightarrow 3y = x + 5 \Rightarrow \boxed{y = \frac{1}{3}x + \frac{5}{3}}$

a)



m_3: slope of $\ell_3 = \frac{1}{3}$

m : slope of $\ell = -3$

$(m \cdot m_3 = -1)$

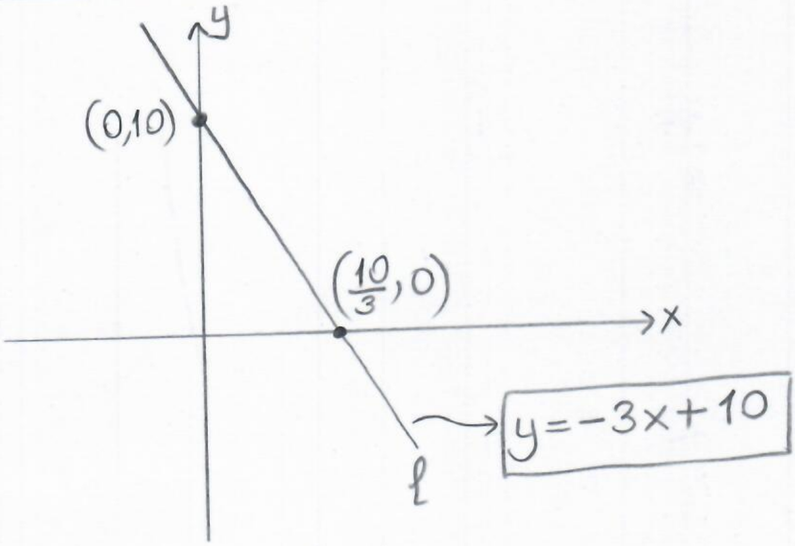
eqn. of ℓ: slope $= -3$, pt.: $(3,1)$ $\Big\}$ $y - 1 = (-3)(x-3) = -3x + 9$

$\boxed{y = -3x + 10}$

b) $x = 0 \Rightarrow y = 10 \longrightarrow (0,10)$: y-intercept of ℓ

$y = 0 \Rightarrow x = \frac{10}{3} \longrightarrow \left(\frac{10}{3}, 0\right)$: x-intercept of ℓ

c)



3. Let $f(x) = -x^2 + 4x - 5$.

a. Find the vertex of $y = f(x)$.

b. Find x-intercepts and y-intercept of $y = f(x)$.

c. Draw the graph of $y = f(x)$ using part a and part b.

d. Draw the graph of $y = f(x)$ using shifting, reflecting and compressing the graph of $y = x^2$.

a) $y = -(x-2)^2 + 4 - 5 = -(x-2)^2 - 1$ $y = -(x-2)^2 - 1$ ∩ concave down parabola

$\boxed{\text{Vertex: } (2,-1)}$

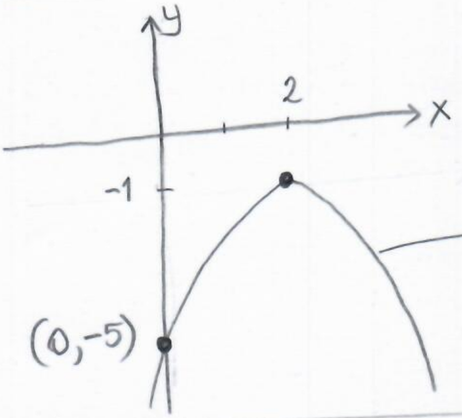
b) $x = 0 \Rightarrow y = -5 \Rightarrow \boxed{(0,-5): \text{y-intercept}}$

$y = 0 \Rightarrow -x^2 + 4x - 5 = 0 \Rightarrow x^2 - 4x + 5 = 0$

$b^2 - 4ac = (-4)^2 - 4(1)(5) = 16 - 20 < 0$

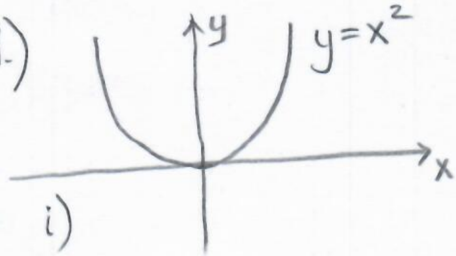
$\Rightarrow$ no real roots $\Rightarrow \boxed{\text{No x-intercepts}}$

c)



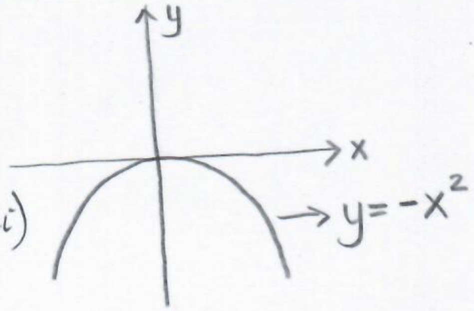
$y = -(x-2)^2 + 1$

d)



$\xrightarrow{\text{reflect}}$ w.r.t. x-axis to get $y = -x^2$

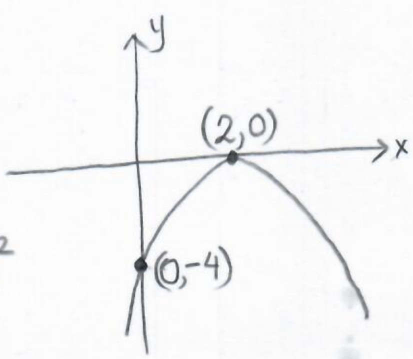
ii)



$y = -x^2$

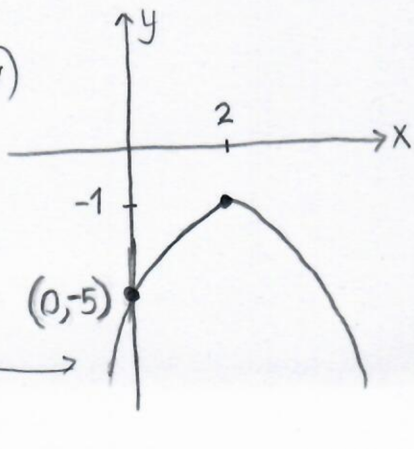
iii)

shift right 2 units to get $y = -(x-2)^2$



shif down 1 unit to get $y = -(x-2)^2 - 1$

iv)



4. Evaluate the following limits

a. $\lim_{x\to 2} \frac{2x^4 - 5x^3 + 5x^2 - 11x + 10}{x^3 - 8} = \frac{2(16)-5(8)+5(4)-11(2)+10}{8-8} = \frac{0}{0}$

$$
\begin{array}{l|l}
2x^4-5x^3+5x^2-11x+10 & x-2 \\ \hline
-2x^4+4x^3 & 2x^3-x^2+3x-5 \\
\quad -x^3+5x^2 & \\
\quad \pm x^3 \mp 2x^2 & \\
\quad\quad 3x^2-11x & \\
\quad\quad -3x^2+6x & \\
\quad\quad\quad -5x+10 & \\
\quad\quad\quad -5x+10 & \\
\quad\quad\quad 0 \quad 0 &
\end{array}
$$

$= \lim_{x\to 2} \frac{\cancel{(x-2)}(2x^3-x^2+3x-5)}{\cancel{(x-2)}(x^2+2x+4)}$

$= \frac{2(8)-4+6-5}{4+4+4} = \boxed{\frac{13}{12}}$

b. $\lim_{x\to 4} \frac{x^3 - 5x^2 + 3x + 4}{\sqrt{x} - 2} = \frac{64-80+12+4}{2-2} = \frac{0}{0}$

$$
\begin{array}{l|l}
x^3-5x^2+3x+4 & x-4 \\ \hline
-x^3 \pm 4x^2 & x^2-x-1 \\
\quad -x^2+3x & \\
\quad + -x^2 \mp 4x & \\
\quad\quad -x+4 & \\
\quad\quad -x+4 & \\
\quad\quad 0 \quad 0 &
\end{array}
$$

$= \lim_{x\to 4} \frac{(x-4)(x^2-x-1)}{(\sqrt{x}-2)} = \frac{0}{0}$

$= \lim_{x\to 4} \frac{\cancel{(\sqrt{x}-2)}(\sqrt{x}+2)(x^2-x-1)}{\cancel{(\sqrt{x}-2)}}$

$= \frac{(2+2)(16-4-1)}{1} = (4)(11) = \boxed{44}$

5. Evaluate the following limits

a. $\lim_{x\to 1} \frac{x^2-3x+2}{\underbrace{x^3-3x^2+3x-1}_{(x-1)^3}} = \frac{1-3+2}{1-3+3-1} = \frac{0}{0}$

$$= \lim_{x\to 1} \frac{\cancel{(x-1)}(x-2)}{\cancel{(x-1)}(x^2-2x+1)} = \frac{1-2}{1-2+1} = \frac{-1}{0} = \boxed{-\infty}$$

$(x\to -\infty)$

b. $\lim_{x\to-\infty} \sqrt{x^2+x+6}+x+2 = \infty - \infty$

$$= \lim_{x\to-\infty} \frac{\left(\sqrt{x^2+x+6}+(x+2)\right)}{\left(\sqrt{x^2+x+6}-(x+2)\right)} \cdot \left(\sqrt{x^2+x+6}-(x+2)\right)$$

$$= \lim_{x\to-\infty} \frac{(x^2+x+6)-(x+2)^2}{-x\left(\sqrt{1+\frac{1}{x}+\frac{6}{x^2}}+1+\frac{2}{x}\right)} = \lim_{x\to-\infty} \frac{\overbrace{\cancel{x^2}+x+6-\cancel{x^2}-4x-4}^{-3x+2}}{-x\left(\sqrt{1+\frac{1}{x}+\frac{6}{x^2}}+1+\frac{2}{x}\right)}$$

$\boxed{\sqrt{x^2}=-x \quad x\to-\infty}$

$$= \lim_{x\to-\infty} \frac{-\cancel{x}\left(3-\frac{2}{x}\right)}{-\cancel{x}\left(\sqrt{1+\frac{1}{x}+\frac{6}{x^2}}+1+\frac{2}{x}\right)} = \frac{3+0}{\sqrt{1-0+0}+1-0} = \boxed{\frac{3}{2}}$$

6. Find the domain of the following functions.

a. $f(x) = \dfrac{\sqrt{16-x^2}}{x^2+x-6} = \dfrac{\sqrt{(4-x)(4+x)}}{(x+3)(x-2)}$

Dom f: $\{x \mid (4-x)(4+x) \geq 0 \text{ and } x \neq -3, x \neq 2\}$

		-4		4	
$x+4$	$-$	0	$+$		$+$
$4-x$	$+$		$+$	0	$-$
$\frac{4-x}{4+x}$	$-$		$+$		$-$

$\frac{4-x}{4+x} \geq 0$ on $[-4, 4]$

$-4 \quad -3 \quad 2 \quad 4$

$\boxed{\text{Dom } f: [-4,-3) \cup (-3,2) \cup (2,4]}$

b. $g(x) = e^{\frac{2}{x+1}} + \ln(x^2 - 2x - 8)$

Dom f: $\{x \mid x \neq -1 \text{ and } (x^2-2x-8) > 0\}$

So that $e^{\frac{2}{x+1}}$ is defined

So that $\ln(x^2-2x-8)$ is defined

$-2 \quad -1 \quad 4$

$x^2-2x-8 = (x-4)(x+2)$

		-2		4	
$x+2$	$-$	0	$+$		$+$
$x-4$	$-$		$-$	0	$+$
$(x-4)(x+2)$	$+$		$-$		$+$

$x^2-2x-8 > 0$ on $(-\infty,-2) \cup (4,\infty)$

$\boxed{\text{So dom } g: (-\infty,-2) \cup (4,\infty)}$

c. $h(x) = \dfrac{x^2-9}{x^2-4x+3} = \dfrac{(x-3)(x+3)}{(x-3)(x-1)} \underset{\substack{\text{if} \\ x \neq 3}}{=} \dfrac{x+3}{x-1}$ which is not defined at $x=1$.

So, Dom h(x): $\mathbb{R} \setminus \{1,3\}$

$\boxed{(-\infty,1) \cup (1,3) \cup (3,\infty)}$