Math 113 ; Nov. 10, 2020

Exercises 5/Q7:

If $\boxed{\ln 2 = 0.7}$ $\boxed{\ln 5 = 1.6}$ $\Rightarrow \log_2 5 = ?$

Change-of-Base Formula: $\boxed{\log_a b = \frac{\ln b}{\ln a}}$ $\left(= \text{or } \frac{\log b}{\log a}\right)$ $\left(= \text{or } \frac{\log_x b}{\log_x a}\right)$

$(a, b \neq 1,\ a, b > 0)$

$x \neq 1$, $x > 0$

$$\log_2 5 = \frac{\ln 5}{\ln 2} = \boxed{\frac{1.6}{0.7}}$$

Ex. 5/5:

$$\frac{1}{6}\left[\ln x - 2(\ln y + 2\ln z)\right] = ?$$

$\ln A - \ln B = \ln\left(\frac{A}{B}\right)$

$\ln A + \ln B = \ln(A \cdot B)$

$\ln A^r = r \ln A$

$$= \frac{1}{6}\left[\ln x - 2(\underbrace{\ln y + \ln z^2}_{\ln(y \cdot z^2)})\right]$$

$$= \frac{1}{6}\left[\ln x - 2\left[\ln(yz^2)\right]\right] = \frac{1}{6}\left[\ln x - \ln(yz^2)^2\right]$$

$$= \frac{1}{6}\ln\left(\frac{x}{(yz^2)^2}\right) = \frac{1}{6}\ln\left(\frac{x}{y^2z^4}\right)$$

$$= \ln\left(\frac{x}{y^2z^4}\right)^{1/6} = \boxed{\ln\left(\frac{\sqrt[6]{x}}{\sqrt[3]{y}\cdot\sqrt[3]{z^2}}\right)}$$

(2)

Ex. 5/9-c) Solve: $\log_2(x-4) + \log_4 3 = \log_8 x$ (*)

$x>0$, $x-4>0 \Rightarrow x>4$

$$\log_4 3 = \frac{\log_2 3}{\log_2 4} = \frac{\log_2 3}{\log_2 2^2} = \frac{\log_2 3}{2\underbrace{\log_2 2}_{1}} = \frac{1}{2}\log_2 3$$

$$\log_8 x = \frac{\log_2 x}{\log_2 8} = \frac{\log_2 x}{\log_2 2^3} = \frac{\log_2 x}{3\underbrace{\log_2 2}_{1}} = \frac{1}{3}\log_2 x$$

(*): $\log_2(x-4) + \frac{1}{2}\log_2 3 = \frac{1}{3}\log_2 x$

$\log_2(x-4) + \log_2(\sqrt{3}) = \log_2(\sqrt[3]{x})$

$\log_2[\sqrt{3}(x-4)] = \log_2(\sqrt[3]{x})$

$\Rightarrow \sqrt{3}(x-4) = \sqrt[3]{x} \Rightarrow (\sqrt{3})^3(x-4)^3 = x$

$[x^3 - 3x^2(4) + 3x(4)^2 - 64] = \frac{x}{3\sqrt{3}}$

$x^3 - 12x^2 + \left(48 - \frac{1}{3\sqrt{3}}\right)x - 64 = 0$

-64	∓ 1
	∓ 2
	∓ 4
	∓ 8
	∓ 16
	∓ 32
	∓ 64

Finding the first root (x_1) of the cubic polyn. by checking the divisors of (-64); we divide the polyn. by $(x-x_1)$ to reduce it to a quadratic

$\Rightarrow$ hence obtain $x_2, x_3 \Rightarrow$ among $\{x_1, x_2, x_3\}$ choose the value(s) s.t. $x>4$.

Ex. 5/10-c) $\log_x\left(\frac{9}{4}\right) = -\frac{2}{3}$

$\log_x\left(\frac{3}{2}\right)^2 = 2 \log_x\left(\frac{3}{2}\right) = -\frac{2}{3}$

$\Rightarrow \log_x\left(\frac{3}{2}\right) = -\frac{1}{3}$

$\log_x y = a \iff x^a = y$

$\Downarrow$

$x^{-1/3} = \frac{3}{2}$

$\frac{1}{\sqrt[3]{x}} = \frac{3}{2} \Rightarrow \boxed{\sqrt[3]{x} = \frac{2}{3}}$

$\Rightarrow \left(\sqrt[3]{x}\right)^3 = \left(\frac{2}{3}\right)^3 \Rightarrow \boxed{x = \frac{8}{27}}$

---

p. 204/4.4 Review Problems/39

$(\log a = \log_{10} a)$

$(a^{\log_a x} = x)$

$\log_a a^x = x$

Simplify $\underbrace{10^{\log x}}_{x} + \underbrace{\log 10^x}_{x \log 10} + \underbrace{\log 10}_{1}$

$\log 10 = 1$

$x + x + 1 = \boxed{2x + 1}$

41.) $\ln y = x^2 + 2 \Rightarrow$ find $y = ?$

$\Rightarrow \boxed{y = e^{x^2+2}}$

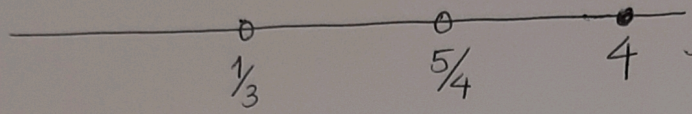
Find x:

45.) $\log(6x-2) = \log(8x-10) \iff \boxed{6x-2 = 8x-10}$

$6x-2 > 0$; $8x-10 > 0$

$6x > 2$; $8x > 10$

$x > \frac{1}{3}$ U $x > \frac{5}{4}$



$\Rightarrow$ Dom. equation $\Rightarrow$ $x > \frac{5}{4}$: $(\frac{5}{4}, \infty)$ ✓

$6x-2 = 8x-10 \Rightarrow 10-2 = 2x \Rightarrow 8 = 2x \Rightarrow \boxed{x=4}$

Soln.: $\{4\}$.

50.) $\ln\left(\frac{x-5}{x-1}\right) = \ln 6 \iff \frac{x-5}{x-1} = 6 \Rightarrow x-5 = 6(x-1)$

$x-5 = 6x-6$

$1 = 5x \Rightarrow \boxed{x = \frac{1}{5}}$

$\frac{x-5}{x-1} > 0$

$x \neq 1$

		1		5	
$x-1$	$-$	0	$+$		$+$
$x-5$	$-$		$-$	0	$+$
$\frac{x-5}{x-1}$	$+$		$-$		$+$

$(-\infty, 1) \cup (5, \infty)$

$x = \frac{1}{5} \in (-\infty, 1) \Rightarrow$ Soln. set: $\left\{\frac{1}{5}\right\}$

51.) $\ln_e(\log_x 3) = 2 \quad x = ?$

$\Leftrightarrow e^2 = \log_x 3 \Rightarrow x^{e^2} = 3 \Rightarrow \boxed{x = (3)^{1/e^2}}$

Ex 3/Q4: $f(x) = \dfrac{1}{2x+3} \Rightarrow \dfrac{f(x+h)-f(x)}{h} = ?$

$$f(x+h) = \frac{1}{2(x+h)+3} = \frac{1}{2x+2h+3}$$

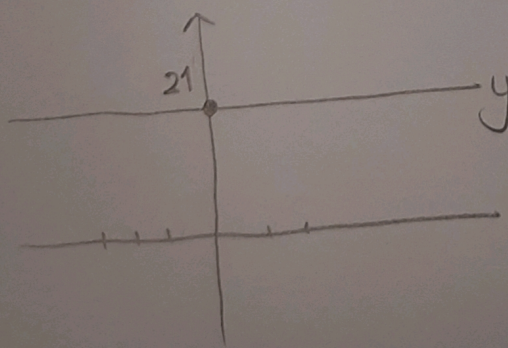
$$f(x+h) - f(x) = \frac{1}{2x+2h+3} - \frac{1}{2x+3} = \frac{(2x+3)-(2x+2h+3)}{(2x+2h+3)(2x+3)}$$

$$= \frac{2x+3-2x-2h-3}{(2x+2h+3)(2x+3)} = \frac{-2h}{(2x+2h+3)(2x+3)}$$

$$\frac{f(x+h)-f(x)}{h} = \frac{-2h}{h(2x+2h+3)(2x+3)} = \boxed{\frac{-2}{(2x+2h+3)(2x+3)}}$$

$(h \neq 0)$

Ex 3/Q2: $\boxed{f(x) = 21} \Rightarrow f(-8) = 21$



$f(21) = 21$

$f(548) = 21$

p.201/29) $\log_4(\underbrace{9x-4}_{>0}) = 2 \iff 4^2 = 9x-4$

$9x > 4$

$x > \frac{4}{9}$

$20 = 9x \Rightarrow \boxed{x = \frac{20}{9}}$ ✓

---

31) $\underbrace{\ln(x-2) + \ln(2x+1)}_{\ln[(x-2)(2x+1)]} = 5$

$\ln[(x-2)(2x+1)] = 5$

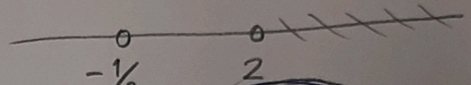
$(2x^2 - 3x - 2) = e^5$

$2x^2 - 3x - 2 - e^5 = 0$

$x_{1,2} = \frac{3 \mp \sqrt{9+4(2)(+2+e^5)}}{2(2)} = \frac{3 \mp \sqrt{9+16+8e^5}}{4}$

$x - 2 > 0 \Rightarrow x > 2$

$2x+1 > 0 \Rightarrow x > -\frac{1}{2}$



$x > 2$

Soln.: $\left\{ \frac{3+\sqrt{25+8e^5}}{4} \right\} \left(> \frac{3+5}{4} = \frac{8}{4} = 2 \right)$

---

33) $\log_2(5x+1) = 4 - \log_2(3x-2)$

$\underbrace{\log_2(5x+1) + \log_2(3x-2)}_{\log_2[(5x+1)(3x-2)]} = 4$

$\log_2[(5x+1)(3x-2)] = 4$

$\iff 2^4 = 15x^2 - 7x - 2 \Rightarrow \boxed{15x^2 - 7x - 18 = 0}$

$x_{1,2} = \frac{7 \mp \sqrt{49-4(15)(-18)}}{2(15)} = \frac{7 \mp \sqrt{1129}}{30}$

$5x+1 > 0 \Rightarrow x > -\frac{1}{5}$

$3x-2 > 0 \Rightarrow x > \frac{2}{3}$

$\Downarrow$

$x > \frac{2}{3}$

$\frac{7+\sqrt{1129}}{30} \cong 1.35 > \frac{2}{3}$

Soln.: $\left\{ \frac{7+\sqrt{1129}}{30} \right\}$

35.) $\log_2\left(\frac{2}{x}\right) = 3 + \log_2 x$ $(x>0)$ ✓

$\underbrace{\log_2 2}_{1} - \log_2 x = 3 + \log_2 x$

$1 - 3 = 2\log_2 x \Rightarrow \frac{-2}{2} = \log_2 x = -1$

$\Rightarrow x = 2^{-1} = \frac{1}{2} \Rightarrow$ Soln. set $\left\{\frac{1}{2}\right\}$

5.) $\ln(\underbrace{-x}_{>0}) = \ln(\underbrace{x^2-6}_{>0})$

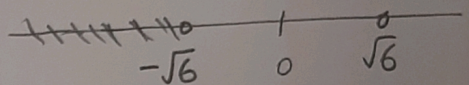
$\Leftrightarrow -x = x^2 - 6 \Rightarrow x^2 + x - 6 = 0$

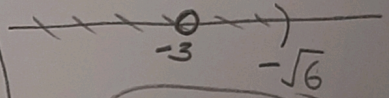
$(x+3)(x-2) = 0$

$x = -3$ or ~~$x = 2$~~ ←

$-x > 0 \Rightarrow x < 0$

$x^2 - 6 > 0 \Rightarrow x^2 > 6 \Rightarrow x > \sqrt{6}$ or $x < -\sqrt{6}$





$x < -\sqrt{6}$

$\Rightarrow$ Soln. set: $\{-3\}$

Contents of Midterm I:

* Intervals, Inequalities, Absolute Value
* Lines, Parabolas
* Functions: Domain and Range, Piecewise Defined Functions, Increasing & Decreasing Functions, Even and Odd Functions
* Functions: Exponential Functions, Logarithmic Functions
* Solving Exponential and Logarithmic Equations