

Exam

Name \_\_\_\_\_

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Provide an appropriate response.

1) By the definition of a derivative, the derivative of  $f(x) = \sqrt{x}$  is

1) \_\_\_\_\_

A)  $\lim_{h \rightarrow 0} \frac{\sqrt{x+h}}{h}$ .

B)  $\lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$ .

C)  $\lim_{h \rightarrow 0} \frac{[\sqrt{x} + \sqrt{h}] - \sqrt{x}}{h}$ .

D)  $\lim_{h \rightarrow 0} \frac{\sqrt{x+h}}{h}$ .

E)  $\lim_{h \rightarrow 0} \frac{[\sqrt{x+h} - \sqrt{x}]}{h}$ .

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

2) By using the definition of a derivative, find  $f'(x)$  where  $f(x) = \frac{2}{3x+5}$ .

2) \_\_\_\_\_

3) Find the slope of the tangent line to the curve  $y = \sqrt{x+2}$  at the point  $(0, \sqrt{2})$ .

3) \_\_\_\_\_

4) Find  $y'$  if  $y = 4x^3 - 6x^2 + 7x - 8$ .

4) \_\_\_\_\_

5) Find  $y'$  if  $y = \frac{7}{3}(9x + 3)$ .

5) \_\_\_\_\_

6) Find  $y'$  if  $y = \frac{1}{4} - \frac{x}{2}$ .

6) \_\_\_\_\_

7) Find  $y'$  if  $y = \frac{3}{4}x^{4/3} - \frac{1}{5}x^{1/4} + x^{-5/6}$ .

7) \_\_\_\_\_

8) Find  $y'$  if  $y = x^{-2} - \sqrt{x} + x^{-4/7}$ .

8) \_\_\_\_\_

9) Find  $y'$  if  $y = \frac{1}{2} - \frac{1}{2x}$ .

9) \_\_\_\_\_

10) Find  $y'$  if  $y = \sqrt{x}(x+2)$ .

10) \_\_\_\_\_

11) Find  $y'$  if  $y = \frac{x^2 + 2x + 3}{\sqrt{x}}$ . 11) \_\_\_\_\_

12) Let  $f(x) = 4x^2 - 3x + 1$ . 12) \_\_\_\_\_  
(a) Find  $f'(x)$ .  
(b) Evaluate  $f'(1)$ .  
(c) Find an equation of the tangent line to the graph of  $y = f(x)$  at the point  $(1, 2)$ .

13) Let  $f(x) = 2x^2 - 6x + 7$ . 13) \_\_\_\_\_  
(a) Find  $f'(x)$ .  
(b) Evaluate  $f'(2)$ .  
(c) Find an equation of the tangent line to the graph of  $y = f(x)$  at the point  $(2, 3)$ .

14) Find all values of  $x$  for which the curve  $y = x^3 + x^2$  has slope 1. 14) \_\_\_\_\_

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

15) An equation of the tangent line to the curve  $y = 4x^2 - 6x - 5$  at the point  $(-1, 5)$  is 15) \_\_\_\_\_  
A)  $y = (8x - 6)(x - 1) + 5$ .  
B)  $y = -14x - 9$ .  
C)  $y = 14x + 71$ .  
D)  $y = -14x + 19$ .  
E)  $y = 8x - 6$ .

16) If  $y = 2^2 + x$ , then  $\frac{dy}{dx} =$  16) \_\_\_\_\_  
A) 1.                      B) 5.                      C) 9.7.                      D)  $2^2$ .                      E) 0.

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

17) Find  $y'$  if  $y = \sqrt[5]{x} \left( \sqrt[3]{\sqrt{x^2}} \right)$  17) \_\_\_\_\_

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

18) A value of  $x$  for which the slope of the curve  $y = \frac{x^3}{3} - \frac{3x^2}{2} + 2x + 1$  is zero is 18) \_\_\_\_\_  
A) 3.                      B) -1.                      C) 2.                      D) 0.                      E) -2.

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

19) Find the rate of change of  $y = x(x^2 + 9x + 3)$  with respect to  $x$ . 19) \_\_\_\_\_

20) Suppose that the equation  $r = 430q - 2q^2$  gives the total revenue  $r$  (in dollars) that a manufacturer receives when  $q$  units of a product are sold. Determine the marginal revenue when  $q = 100$  and interpret your result. 20) \_\_\_\_\_

21) Find the rate of change of  $y$  with respect to  $x$  when  $y = 7 - 3x + 11x^3$ . 21) \_\_\_\_\_

22) Find  $y'$  if  $y = (x^3 - 10x + 2)(x^2 + 7x + 1)$ . 22) \_\_\_\_\_

23) Find  $y'$  if  $y = \frac{2x + 3}{7 - 5x}$ . 23) \_\_\_\_\_

24) Find  $y'$  if  $y = \frac{x^2 + 1}{2x^3 - 1}$ . 24) \_\_\_\_\_

25) Find the slope of the curve  $y = \frac{4x}{x + 2}$  at the point where  $x = 3$ . 25) \_\_\_\_\_

26) Find the slope of the curve  $y = \frac{2x + 5}{x - 3}$  at the point  $(4, 13)$ . 26) \_\_\_\_\_

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

27) If  $f(x) = \frac{x^2 + 4}{x^2 - 2}$ , then  $f'(x) =$  27) \_\_\_\_\_

A)  $\frac{12x}{(x^2 + 2)^2}$

B) 1

C)  $\frac{6}{(x^2 - 2)^2}$

D)  $\frac{4x + 26x - 9x^2}{(3x + 2)^2}$

E)  $-\frac{12x}{(x^2 - 2)^2}$

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

28) 28) \_\_\_\_\_

Find the derivative of  $y = \frac{\frac{1}{x} - \frac{7x}{x^2 + 1}}{\frac{2}{x} - \frac{3x}{x^2 + 1}}$ .

29) Differentiate:  $f(x) = (3x^2 + x)(8x - 7)$  29) \_\_\_\_\_

30) Differentiate:  $h(x) = (4x - 5)\sqrt{x}$  30) \_\_\_\_\_

31) Differentiate:  $g(x) = (x - 8)(2x - 7)(x + 5)$  31) \_\_\_\_\_

32) Differentiate:  $g(x) = \frac{3\sqrt{x}}{x + 4}$  32) \_\_\_\_\_

33) Differentiate:  $f(x) = \frac{3x^2 + 6}{5x^2 - 2x}$

33) \_\_\_\_\_

# Answer Key

Testname: UNTITLED3

- 1) B
- 2)  $-\frac{6}{(3x+5)^2}$
- 3)  $\frac{1}{2\sqrt{2}}$
- 4)  $12x^2 - 12x + 7$
- 5) 21
- 6)  $-\frac{1}{2}$
- 7)  $x^{1/3} - \frac{1}{20}x^{-3/4} - \frac{5}{6}x^{-11/6}$
- 8)  $-2x^{-3} - \frac{1}{2\sqrt{x}} - \frac{4}{7}x^{-11/7}$
- 9)  $\frac{1}{2x^2}$
- 10)  $\frac{3}{2}x^{1/2} + \frac{1}{x^{1/2}}$
- 11)  $\frac{3}{2}x^{1/2} + x^{-1/2} - \frac{3}{2}x^{-3/2}$
- 12) (a)  $8x - 3$                       (b) 5                      (c)  $y = 5x - 3$
- 13) (a)  $4x - 6$                       (b) 2                      (c)  $y = 2x - 1$
- 14)  $-1, \frac{1}{3}$
- 15) B
- 16) A
- 17)  $\frac{13}{15}x^{-2/15}$
- 18) C
- 19)  $3x^2 + 18x + 3$
- 20) 30; if the number of units sold increases from 100 to 101, then the total revenue increases by approximately \$30.
- 21)  $33x^2 - 3$
- 22)  $(x^3 - 10x + 2)(2x + 7) + (x^2 + 7x + 1)(3x^2 - 10) = 5x^4 + 28x^3 - 27x^2 - 136x + 14$
- 23)  $\frac{29}{(7-5x)^2}$
- 24)  $\frac{-2x(x^3 + 3x + 1)}{(2x^3 - 1)^2}$
- 25)  $\frac{8}{25}$
- 26) -11
- 27) E
- 28)  $\frac{-22x}{(2-x^2)^2}$
- 29)  $f'(x) = 8(3x^2 + x) + (8x - 7)(6x + 1)$  or  
 $f'(x) = 72x^2 - 26x - 7$

## Answer Key

Testname: UNTITLED3

$$30) h'(x) = \frac{4x - 5}{2\sqrt{x}} + 4\sqrt{x} \text{ or}$$

$$h'(x) = \frac{12x - 5}{2\sqrt{x}}$$

$$31) g'(x) = (2x - 7)(x + 5) + 2(x - 8)(x + 5) + (x - 8)(2x - 7) \text{ or}$$

$$g'(x) = 8x^3 - 252x^2 + 104x + 28$$

$$\frac{3(x + 4)}{2\sqrt{x}} - 3\sqrt{x}$$

$$32) g'(x) = \frac{\frac{3(x + 4)}{2\sqrt{x}} - 3\sqrt{x}}{(x + 4)^2} \text{ or}$$

$$g'(x) = \frac{-3x + 12}{2\sqrt{x}(x + 4)^2}$$

$$33) f'(x) = \frac{6x(5x^2 - 2x) - (3x^2 + 6)(10x - 2)}{(5x^2 - 2x)^2} \text{ or}$$

$$f'(x) = \frac{20x^3 - 6x^2 - 60x + 12}{(5x^2 - 2x)^2}$$