

Exam

Name _____

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

1) Let $f(x)$ be defined by 1) _____

$$f(x) = \begin{cases} \frac{x^2 - 9}{x - 3} & \text{if } x \neq 3 \\ 6 & \text{if } x = 3 \end{cases}$$

Where, if anywhere, is f discontinuous?

- A) at $x = 0$ B) nowhere C) at $x = 6$ D) at $x = 3$ E) at $x = -3$

TRUE/FALSE. Write 'T' if the statement is true and 'F' if the statement is false.

2) True or False: If $h(x) = \frac{x^2 + 2x - 3}{x - 1}$ if $x \neq 1$ and $h(1) = 4$, then h is continuous at every x . 2) _____

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

3) Let $f(x)$ have values 3) _____

$$f(x) = \begin{cases} -2 & \text{if } x \leq -5 \\ \frac{2}{5}x & \text{if } -5 < x \leq 5 \\ 2 & \text{if } x > 5 \end{cases}$$

Where is f discontinuous?

- A) at both $x = -5$ and $x = 5$
B) nowhere
C) at $x = -5$ only
D) at $x = 5$ only
E) none of the above

4) For what value of the constant c is the function $f(x) = \begin{cases} x + c & \text{if } x < 2 \\ cx^2 + 1 & \text{if } x \geq 2 \end{cases}$ 4) _____

continuous at everywhere?

- A) $\frac{1}{2}$
B) $\frac{1}{3}$
C) 0
D) 1
E) none of the above

- 5) Let $f(x)$ represent: 5) _____
- $$f(x) = \begin{cases} x^2 + 2, & \text{for } x > 5 \\ 24, & \text{for } x = 5 \\ 4x + 7, & \text{for } x < 5 \end{cases}$$

Where is $f(x)$ discontinuous?

- A) 0
 B) 2
 C) 4
 D) 5
 E) none of the above

- 6) For what values of the constants c and k is the function 6) _____

$$f(x) = \begin{cases} x^3 + k & \text{if } -1 \leq x \leq 3 \\ cx & \text{if } x < -1 \text{ or } x > 3 \end{cases}$$

continuous at all x ?

- A) $c = 7, k = -6$
 B) $c = 6, k = 7$
 C) $c = -7, k = 6$
 D) $c = -6, k = -7$
 E) $c = 7, k = -6$

- 7) Find the equation of the tangent line to the curve $y = 2x - x^2$ at the point $(2, 0)$. 7) _____

- A) $2x - y - 4 = 0$
 B) $2x - y + 4 = 0$
 C) $2x + y + 4 = 0$
 D) $2x + y - 4 = 0$
 E) $2x + y = 0$

- 8) If the line $4x - 9y = 0$ is tangent in the first quadrant to the graph of $y = \frac{1}{3}x^3 + c$, what is the value of 8) _____

c ?

- A) $\frac{16}{81}$ B) $\frac{81}{16}$ C) $-\frac{16}{81}$ D) $\frac{18}{81}$ E) $\frac{1}{81}$

- 9) Using the definition of the derivative find, the derivative of $f(x) = \sqrt{x+2}$. 9) _____

- A) $\frac{1}{\sqrt{x+2}}$ B) $\frac{3}{2\sqrt{x+2}}$ C) $\frac{2}{2\sqrt{x+2}}$ D) $\frac{1}{2\sqrt{x+2}}$ E) $\frac{1}{2\sqrt{x-2}}$

- 10) Find the tangent line to the curve $y = \frac{x}{4-x}$ at the origin. 10) _____

- A) $y = -\frac{1}{4}x$ B) $y = -\frac{1}{2}x$ C) $y = \frac{1}{4}x$ D) $y = x$ E) $y = \frac{1}{2}x$

- 11) If $f(x) = \frac{4}{5}(\sqrt{9-x})$, calculate $f'(5)$ by using the definition of the derivative. 11) _____

- A) $-\frac{1}{5}$ B) $-\frac{1}{10}\sqrt{5}$ C) $-\frac{4}{5}$ D) $\frac{2}{5}$ E) $\frac{1}{5}$

- 12) Find the slope of the line tangent to the curve $x^3 y = 1$ at the point $\left(3, \frac{1}{27}\right)$. 12) _____
- A) $\frac{1}{27}$ B) $-\frac{1}{27}$ C) $\frac{1}{9}$ D) $-\frac{2}{27}$ E) $\frac{2}{27}$

- 13) Find the derivative of the function $f(x) = \frac{x^2 + x - 2}{x^3 + 6}$. 13) _____
- A) $\frac{-x^4 + 2x^3 + 6x^2 + 12x + 6}{(x^3 + 6)^2}$
- B) $\frac{-x^4 - 2x^3 - 6x^2 + 12x + 6}{(x^3 + 6)^2}$
- C) $\frac{2x + 1}{3x^2}$
- D) $\frac{-x^4 - 2x^3 + 6x^2 + 12x + 6}{(x^3 + 6)^2}$
- E) $\frac{-x^4 - 2x^3 + 6x^2 + 12x - 6}{(x^3 + 6)^2}$

- 14) Calculate $f'(2)$ if $f(x) = \frac{x^2 + 3x + 2}{x^2 - 3x + 4}$. 14) _____
- A) 2 B) 7 C) $\frac{1}{4}$ D) 4 E) $\frac{1}{2}$

- 15) Find the derivative of $f(x) = \frac{1}{(3x^2 + 5)^4}$. 15) _____
- A) $-\frac{24x}{(3x^2 + 5)^5}$
- B) $-\frac{4}{(3x^2 + 5)^5}$
- C) $-\frac{12x}{(3x^2 + 5)^3}$
- D) $\frac{24x}{(3x^2 + 5)^5}$
- E) $\frac{12x}{(3x^2 + 5)^3}$

16) Differentiate the following function: $f(x) = \left(\frac{3x - 1}{x^2 + 3}\right)^2$. 16) _____

A) $\frac{2(3x - 1)(-3x^2 + 2x - 9)}{(x^2 + 3)^3}$

B) $\frac{3(3x - 1)(-3x^2 + 2x + 9)}{(x^2 + 3)^3}$

C) $\frac{2(3x - 1)(-3x^2 + 2x + 9)}{(x^2 + 3)^3}$

D) $\frac{2(3x - 1)(3x^2 + 2x + 9)}{(x^2 + 3)^3}$

E) none of the above

17) Find an equation of the line tangent to the curve $y = (x^3 + 2)^9$ at the point $(-1, 1)$. 17) _____

A) $27y + x - 26 = 0$

B) $27x - y + 28 = 0$

C) $9x - y + 10 = 0$

D) $27x + y + 26 = 0$

E) $27y - x - 28 = 0$

18) Find $f'''(x)$ if $f(x) = \pi x^3 - 7x$. 18) _____

A) $6\pi x$

B) $6\pi x - 7$

C) 6π

D) π

E) 0

19) Find all local extreme values of the function $f(x) = 2x^3 + 3x^2 - 12x + 13$ and their locations. 19) _____

A) local maximum 33 at $x = -2$, local minimum 26 at $x = 1$

B) local maximum 26 at $x = -1$, local minimum 17 at $x = 2$

C) local maximum 17 at $x = -1$, local minimum 26 at $x = 2$

D) local maximum 26 at $x = -2$, local minimum 33 at $x = 1$

E) no local extrema

20) Find all local extreme values of the function $f(x) = x^3 - 6x^2 + 12x - 5$ and their locations. 20) _____

A) local maximum -61 at $x = -2$, local minimum 3 at $x = 2$

B) local maximum 3 at $x = -2$, local minimum -61 at $x = 2$

C) local maximum -61 at $x = 2$, local minimum 3 at $x = -2$

D) local maximum 3 at $x = 2$, local minimum -61 at $x = -2$

E) no local extrema

21) Determine the concavity of $f(x) = x^3 - 24x^2 + 6x + 18$ and identify any points of inflection. 21) _____

A) concave downwards on $(-\infty, 8)$, upwards on $(8, \infty)$; inflection at $x = 8$

B) concave upwards on $(-\infty, 8)$, downwards on $(8, \infty)$; inflection at $x = 8$

C) concave downwards on $(-\infty, -8)$, upwards on $(-8, \infty)$; inflection at $x = -8$

D) concave upwards on $(-\infty, -8)$, downwards on $(-8, \infty)$; inflection at $x = -8$

E) concave upwards on $(-\infty, \infty)$; no inflection points

- 22) Find the concavity and inflection point(s) of the function $f(x) = 7 - 6x^2 - 2x^3$. 22) _____
- A) concave up on $(1, \infty)$, concave down on $(-\infty, 1)$; inflection at $x = 1$
 B) concave up on $(-1, \infty)$, concave down on $(-\infty, -1)$; inflection at $x = -1$
 C) concave down on $(-1, \infty)$, concave up on $(-\infty, -1)$; inflection at $x = -1$
 D) concave down on $(1, \infty)$, concave up on $(-\infty, 1)$; inflection at $x = 1$
 E) concave up on $(-\infty, \infty)$

- 23) What are the asymptotes of the graph of $y = \frac{2x^2 - 3}{x^2 - x - 2}$? 23) _____
- A) horizontal asymptote at $y = 2$, vertical asymptotes at $x = 1$ and $x = -2$
 B) horizontal asymptote at $y = 2$, vertical asymptotes at $x = -1$ and $x = 2$
 C) horizontal asymptote at $y = \sqrt{\frac{3}{2}}$, vertical asymptotes at $x = 1$ and $x = 2$
 D) oblique asymptote at $y = x - 2$, vertical asymptotes at $x = -1$ and $x = 2$
 E) oblique asymptote at $y = -x - 2$, vertical asymptotes at $x = -1$ and $x = 2$

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

- 24) Use information obtained from f and its first two derivatives to sketch the graph of the function $f(x) = x^3 - 2x^2 - 4x + 3$. 24) _____
- 25) Find the local extrema and inflection points of the function $f(x) = (x^2 - 1)^2$ and sketch its graph. 25) _____

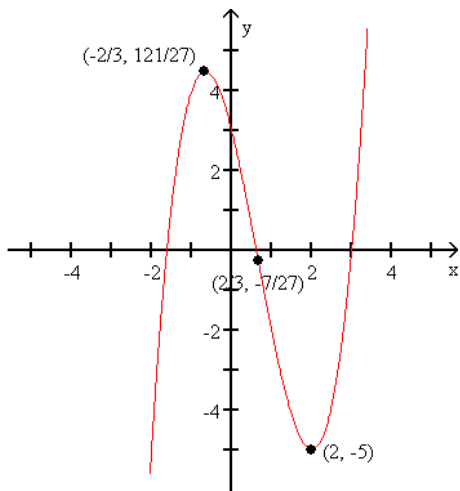
MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

- 26) Find two positive numbers whose sum is 16 such that the product of one number and the cube of the other number is a maximum. 26) _____
- A) 3 and 13 B) 9 and 7 C) 1 and 15 D) 4 and 12 E) 8 and 8
- 27) Find two nonnegative numbers whose sum is 9 such that the sum of one number and the square of the other number is a maximum. 27) _____
- A) 1 and 8
 B) 4 and 5
 C) 3 and 6
 D) $1/2$ and $17/2$
 E) 0 and 9

Answer Key

Testname: UNTITLED1

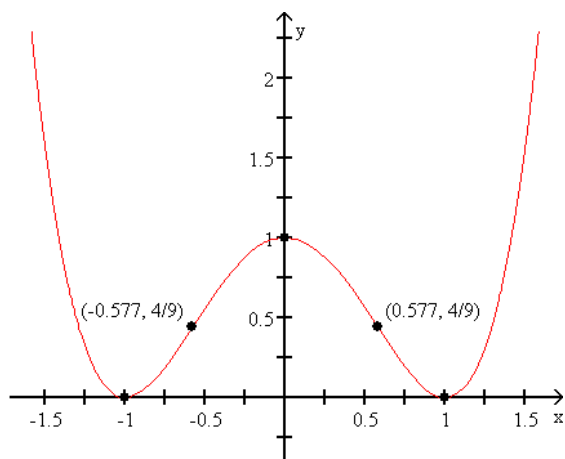
- 1) B
- 2) TRUE
- 3) B
- 4) B
- 5) D
- 6) E
- 7) D
- 8) A
- 9) D
- 10) C
- 11) A
- 12) B
- 13) D
- 14) E
- 15) A
- 16) C
- 17) B
- 18) C
- 19) A
- 20) E
- 21) A
- 22) C
- 23) B
- 24) Local max $(-2/3, 121/27)$, local min $(2, -5)$, inflection point $(2/3, -7/27)$.



Answer Key

Testname: UNTITLED1

25) Local max at $(0,1)$, local min at $(\pm 1,0)$, inflections at $(\pm 1/\sqrt{3}, 4/9)$



26) D

27) E