Ch.6: Derivatives:

$$\frac{d}{dx}(x^n) = n x^{n-1}$$

$$\frac{d}{dx}\left[f(x) \mp g(x)\right] = f'(x) \mp g'(x)$$

$$\frac{d}{dx}\left[f(x) \cdot g(x)\right] = f' \cdot g + g' \cdot f$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f' \cdot g - g' \cdot f}{g^2}$$

$$\frac{d}{dx}(c f) = c f'$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x} \quad \left[\frac{d}{dx}(\log_a x) = \frac{d}{dx}\left(\frac{\ln x}{\ln a}\right) = \frac{1}{\ln a} \cdot \frac{1}{x}\right]$$

p.49/6.14 $f(x) = x^2 \cdot \ln(x^3) = x^2 \cdot (3\ln x)$

$$f'(x) = (2x)(3\ln x) + (x^{\cancel{2}})\left(3 \cdot \frac{1}{\cancel{x}}\right)$$

$$= 6x\ln x + 3x$$

$$= 3x\left[2\ln x + 1\right]$$

$$= \boxed{3x\left[\ln x^2 + 1\right]}$$

6.24 $f(x) = \dfrac{2 - 3\ln x}{5\ln x + 1}$

$$f'(x) = \frac{\left(-3\frac{1}{x}\right)(5\ln x + 1) - \left(\frac{5}{x}\right)(2 - 3\ln x)}{(5\ln x + 1)^2}$$

$$= \frac{\cancel{-\frac{15}{x}\ln x} - \frac{3}{x} - \frac{10}{x} + \cancel{\frac{15}{x}\ln x}}{(5\ln x + 1)^2}$$

$$= \boxed{\frac{-13}{x \cdot (5\ln x + 1)^2}}$$

Ch.7: Chain rule:

$$\boxed{\left[f(g(x))\right]' = f'(g(x)) \cdot g'(x)}$$

$$u = g(x) \Rightarrow f(g(x)) = f(u)$$

$$\boxed{\frac{df}{dx} = \frac{df}{du} \cdot \frac{du}{dx}}$$

Ex.7.1: $\dfrac{d}{dx}(3x^2+1)^5 = ?$

$$= 5(3x^2+1)^4 \cdot (6x) = 30x\,(3x^2+1)^4$$

Ex.7.2: $f(x) = e^{x^5}$

$$f'(x) = e^{x^5} \cdot 5x^4 = 5x^4 \cdot e^{x^5}$$

Ex.7.3: $f(x) = \ln(1 + 2x + 5x^2)$

$$f'(x) = \frac{1}{1 + 2x + 5x^2}(2 + 10x)$$

$$\frac{d}{dx}\left(e^{u(x)}\right) = e^{u(x)} \cdot u'(x)$$

$$\frac{d}{dx}\ln(u(x)) = \frac{1}{u(x)} \cdot u'(x)$$

$$\boxed{\frac{d}{dx}\left(a^{u(x)}\right) = a^{u(x)} \cdot u'(x) \cdot \ln a}$$

$$\frac{d}{dx}\left(\log_a u(x)\right) = \frac{u'(x)}{u(x)} \frac{1}{\ln a}$$

$a > 0,\ a \neq 1$

Ex.7.4: $f'(x) = ?$ HW

a.) $f(x) = \sqrt{2x - 3} = (2x - 3)^{1/2}$

b.) $f(x) = (x^3 + e^x)^7$

c.) $f(x) = \ln\left(\dfrac{x+1}{2x+1}\right) = \ln(x+1) - \ln(2x+1)$

Ex.7.5:

a) $f(x)=e^{ax} \Rightarrow f'(x)=e^{ax}\cdot(a)=ae^{ax}$

b) $f(x)=\ln(ax)=\frac{1}{\cancel{a}x}\cdot\cancel{a}=\frac{1}{x}$

or $(\ln a+\ln x)'=0+\frac{1}{x}=\frac{1}{x}$

c) $f(x)=e^{x^2-x} \Rightarrow f'(x)=e^{x^2-x}\cdot(2x-1)$

$\Rightarrow \boxed{f'(x)=(2x-1)e^{x^2-x}}$

d) $f(x)=\ln(x^8)$

$f'(x)=\frac{1}{x^8}\cdot 8x^7=\boxed{\frac{8}{x}}$

or $f(x)=8\ln x$

$f'(x)=8\cdot\frac{1}{x}=\boxed{\frac{8}{x}}$

Logarithmic diff.:

eg. $y=\frac{(x^3+1)(x^2-5)}{(x^4-5x^2+7x-8)(x^3-5x+3)} \Rightarrow y'=?$

$\ln y=\ln(x^3+1)+\ln(x^2-5)-\ln(x^4-5x^2+7x-8)-\ln(x^3-5x+3)$

diff. both sides:

$$\frac{y'}{y}=\frac{3x^2}{x^3+1}+\frac{2x}{x^2-5}-\frac{4x^3-10x+7}{x^4-5x^2+7x-8}-\frac{3x^2-5}{x^3-5x+3}$$

$\Rightarrow y'=y\Big[\qquad\qquad\Big]$

$* \; f(x)=\underbrace{a^{u(x)}}_{y} \Rightarrow f'(x)=?$

$\ln y=\ln(a^{u(x)})=(u(x))(\ln a)$

$\frac{d}{dx} \Rightarrow \frac{y'}{y}=(\ln a)\,u'(x) \Rightarrow \boxed{y'=a^{u(x)}\cdot u'(x)\cdot \ln a}$

$* \; g(x)=\log_a u(x) \Rightarrow g'(x)=?$

$g(x)=\frac{\ln(u(x))}{\ln a} \Rightarrow \boxed{g'(x)=\frac{1}{\ln a}\cdot\frac{u'(x)}{u(x)}}$

Ex.7.6: $y=x^x \Rightarrow \ln y=\ln x^x=(x)(\ln x)$

$\frac{y'}{y}=(1)(\ln x)+(x)\left(\frac{1}{x}\right)=(\ln x+1)$

$\Rightarrow y'=y(\ln x+1)=\boxed{x^x(\ln x+1)}$

Ex.7.7: $y=x^{\ln x} \Rightarrow \ln y=\ln(x^{\ln x})=\underbrace{(\ln x)(\ln x)}_{(\ln x)^2}$

$\Rightarrow \underbrace{\frac{y'}{y}}_{\text{chainrule}}=2(\ln x)\cdot\left(\frac{1}{x}\right)=\frac{2\ln x}{x}$

$$\boxed{y'=y\left(\frac{2\ln x}{x}\right)=\left(x^{\ln x}\right)\left(\frac{2\ln x}{x}\right)}$$

Ex.7.8: $y=(x+e^x)^{\ln x}$, $y'=?$

$$\ln y = \ln (x+e^x)^{\ln x} = (\ln x)(\ln(x+e^x))$$

$$\frac{y'}{y} = \left(\frac{1}{x}\right)(\ln(x+e^x)) + (\ln x)\left[\frac{1}{x+e^x}\cdot(1+e^x)\right]$$

$$\frac{y'}{y} = \frac{\ln(x+e^x)}{x} + \frac{(1+e^x)(\ln x)}{x+e^x}$$

$$\Rightarrow \boxed{y' = (x+e^x)^{\ln x}\cdot\left[\frac{\ln(x+e^x)}{x} + \frac{(1+e^x)(\ln x)}{x+e^x}\right]}$$

p.54 7.10 $f(x)=\sqrt{x^2+2e^{3x}} \Rightarrow f'=?$

$$=(x^2+2e^{3x})^{1/2}$$

$$\Rightarrow f'(x) = \frac{1}{2}(x^2+2e^{3x})^{-1/2}\cdot(2x+2.e^{3x}.3)$$

$$= \frac{\cancel{2}(x+3e^{3x})}{\cancel{2}\sqrt{x^2+2e^{3x}}} = \boxed{\frac{x+3e^{3x}}{\sqrt{x^2+2e^{3x}}}}$$

7.12) $f(x)=xe^x\log_3(x+x^4)$

$$f'(x)=(1)(e^x)(\log_3(x+x^4))+(x)(e^x)\left(\frac{1+4x^3}{x+x^4}\cdot\frac{1}{\ln 3}\right)$$

$$= e^x\log_3(x+x^4) + \frac{xe^x(1+4x^3)}{(\ln 3)(x+x^4)}$$

Find $f''(x)=?$ $\boxed{(a^{u(x)})' = a^{u(x)}\cdot u'(x).\ln a}$

7.13) $f(x)=5^{2x}$ $\ln 5^2 = \ln 25$

$$f'(x)=5^{2x}\cdot(2).\ln 5 = (2\ln 5)(5^{2x})$$

$$f''(x)=(\ln 25)\left[5^{2x}\cdot(2).\ln 5\right]$$

$$=(2\ln 5)(\ln 25)(5^{2x})$$

$$=\boxed{(\ln 25)^2\cdot 5^{2x}}$$

7.19 $f(x)=(\ln x)^x = y$

$$\ln y = \ln(\ln x)^x = [x][\ln(\ln x)]$$

$$\frac{y'}{y} = (1)(\ln(\ln x)) + (x)\left(\frac{1}{\ln x}\cdot\frac{1}{x}\right)$$

$$= \ln(\ln x) + \frac{1}{\ln x}$$

$$\Rightarrow \boxed{y'} = y\left(\ln(\ln x)+\frac{1}{\ln x}\right) = \boxed{(\ln x)^x\cdot\left[\ln(\ln x)+\frac{1}{\ln x}\right]}$$

7.22 Find the equation of the tangent line to $f(x)$ at $x_0=2$

$f(x)=x^2(1-x)^2$ $\quad y_0=f(2)=4(1-2)^2=4$

slope $= m = f'(x_0) = f'(2)$

$$f'(x)=(2x)((1-x)^2)+(x^2)(2(1-x)\cdot(-1))$$

$$=2x(1-x)[1-x-x]$$

$$f'(x)=2x(1-x)(1-2x)$$

$$f'(2)=2(2)(1-2)(1-4)=(-4)(-3)=\boxed{12}$$

tg. line: $y-4=(12)(x-2) \Rightarrow \boxed{y=12x-20}$